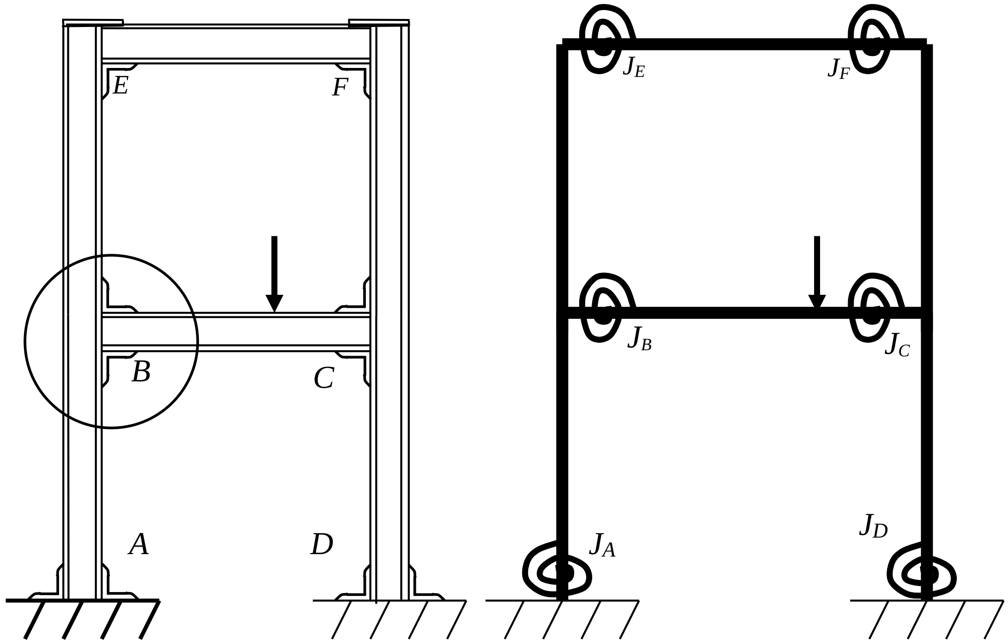
# 39 MULTI-STOREY BUILDINGS – III

## 1.0 INTRODUCTION

In the earlier two chapters, the analysis and design of the multi-storey building frames were illustrated based on the assumptions that all members meeting at a particular joint of the structure undergo the same amount of rotation and hence the name *'rigid framed structures'*. In other words, the joints are assumed to be "rigid", and there is no relative rotation of one member with respect to the other. In fact, this has been the main underlying assumption in most of our frame analysis. At the other extreme, we assume the joints to be hinged in the case of truss structures. Thus, at the supports of steel structures, it is assumed that either ideally fixed or ideally pinned conditions exist. In reality, many "rigid" connections in steel structures permit a certain amount of rotation to take place within the connections, and most "pinned" connections offer a small amount of restraint against rotation. Thus, if a more accurate analysis of such structures is desired, it is necessary to consider the connections as being flexible or semi – rigid.

## 2.0 CONNECTION FLEXIBILITY IN STEEL FRAMES

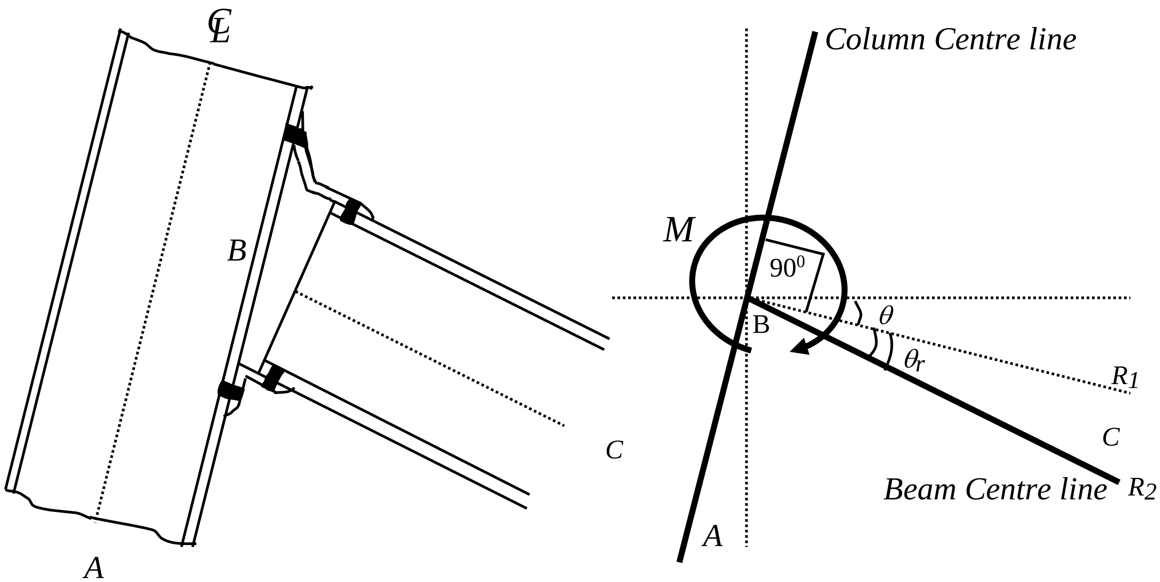
To illustrate the connection flexibility in steel frames, let us consider the two-storey steel frame structure shown in Fig.1a. The beam *BC* is connected to the supporting columns by connections which may be carried out in several ways.




*(a) Actual frame* *(b) Idealised frame*

***Fig. 1 Steel frame connections and their modelling***

For example, the ends of the beam may be welded directly to the column flanges, or by using angles attached to the top and bottom of the beam, or framing angles may be used on the web of the beam. Regardless of the manner of connection, there will be a certain amount of flexibility in connections due to the deformations of the connection components and the flanges of the column. For this illustration let us assume that the beam column joint at *B* in Fig.1(a) is made up of using '*top angle and seat angle*' connection. To understand the connection flexiblity, let us focus our attention at the deformation of joint *B*, due to application of load. The deformation of the joint *B*, to an exaggerated scale, is shown in Fig.2(a). From this figure it is inferred that as the moment to be transferred increases, the connection angles are likely to deform.




(*a*) *Deformed shape of joint B (Exaggerated scale)*

*(b) Centre line diagram of joint B*

***Fig.2 Connection flexibility of beam-column joint***

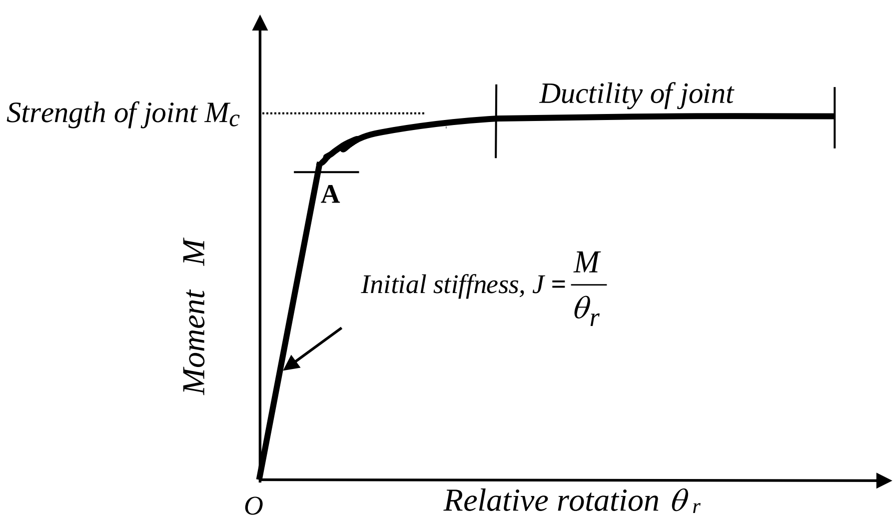



***Fig.3 Joint characteristics (M-$\theta_r$ relationship)***

Due to this connection deformation, the beam *BC* will rotate through a larger clockwise angle than the column *BA*. From Fig.2(b), we infer that if the beam column joint were to be perfectly rigid, the beam *BC* would have rotated along the line $BR_1$ which is orthogonal to the deformed column centre line. Instead, the beam has rotated to the position along $BR_2$ . This means that the beam has rotated an extra angle $\theta_r$ relatively to the column, called the '*relative angle of rotation*'. It is obvious that the rotation component $\theta_r$ is due to the connection flexibility. Hence if one wants to consider connection flexibility in the analysis, the relation between the applied moment '*M*' at the joint and the relative rotation $\theta_r$ becomes very important. Fig.3 shows a typical *M* vs $\theta_r$ relation observed in flexible connections. Initially the connection behaves nearly elastically and the curve *OA* is nearly a straightline with a slope $J=M/\theta_r$, which represents the rotational spring constant of the connection and is called the *joint modulus*. On further loading, the joint begins to deform inelastically and the angle of rotation increases rapidly. The connection stiffness decreases as the load increases and it is characterised by the $M$-$\theta_r$ curve becoming flatter and flatter as it asymptotically approaches the plastic moment capacity $M_c$ of the connection. Due to inherent ductility in the connection components, usually there would be considerable amount of ductility in the joints. However at normal working loads, the behavior of the connections of most structures can be approximated by a straightline such as *OA*. For future discussions of this chapter we would assume that connections behave linearly and their stiffnesses could be represented by their joint modulus *'J'*. In such a case, we can idealise the steel frame in Fig.1(a) as composed of members with an elastic rotational springs located at connections joining beam and column. Such an idealised frame is shown in Fig.1(b). For clarity in drawing the sketches, the springs are located at a small distance from the corresponding joints of the structure. For example, the hinge and rotational spring representing the connection at joint *B* are located at a small distance from the theoritical intersection of members *BA* and *BC* in Fig.1(b). In calculations it will be assumed that this distance is equal to zero, although the hinge and spring are still considered to be a part of beam *BC*.

## 3.0 MOMENT – ROTATION CHARACTERISTICS OF STRUCTURAL STEEL CONNECTIONS

The various types of structural steel connections that are commonly used in practice are shown in Fig.4. Depending on the flexibility (or inversely the stiffness of the connections) the various type connections can be classified into flexible or stiff connections. The schematic classification of these connections has been presented in Fig. 5(a). For ease of design these connections are better classified as rigid (in which the rigid elastic assumption is valid), semi-rigid (in which connection flexibility is to be taken into account) and pinned connections (in which no moment is assumed to be transferred across the joint). As evident from the complexity of connections shown in Fig.4, it is almost impossible to develop analytical expressions for calculating the stiffness of these connections. Hence, connection characteristics are mostly determined using experiments. Based on these experiments, analytical expressions are prescribed for design in the form of empirical equations. To get the empirical equations, numerous results of investigations of semi-rigid connections are put into a data bank of $M$- $\theta_r$ curves. Then curve-fitting methods are used on the experimental data to develop appropriate $M$-$\theta_r$ curves for design. There are several curve-fitting techniques used.

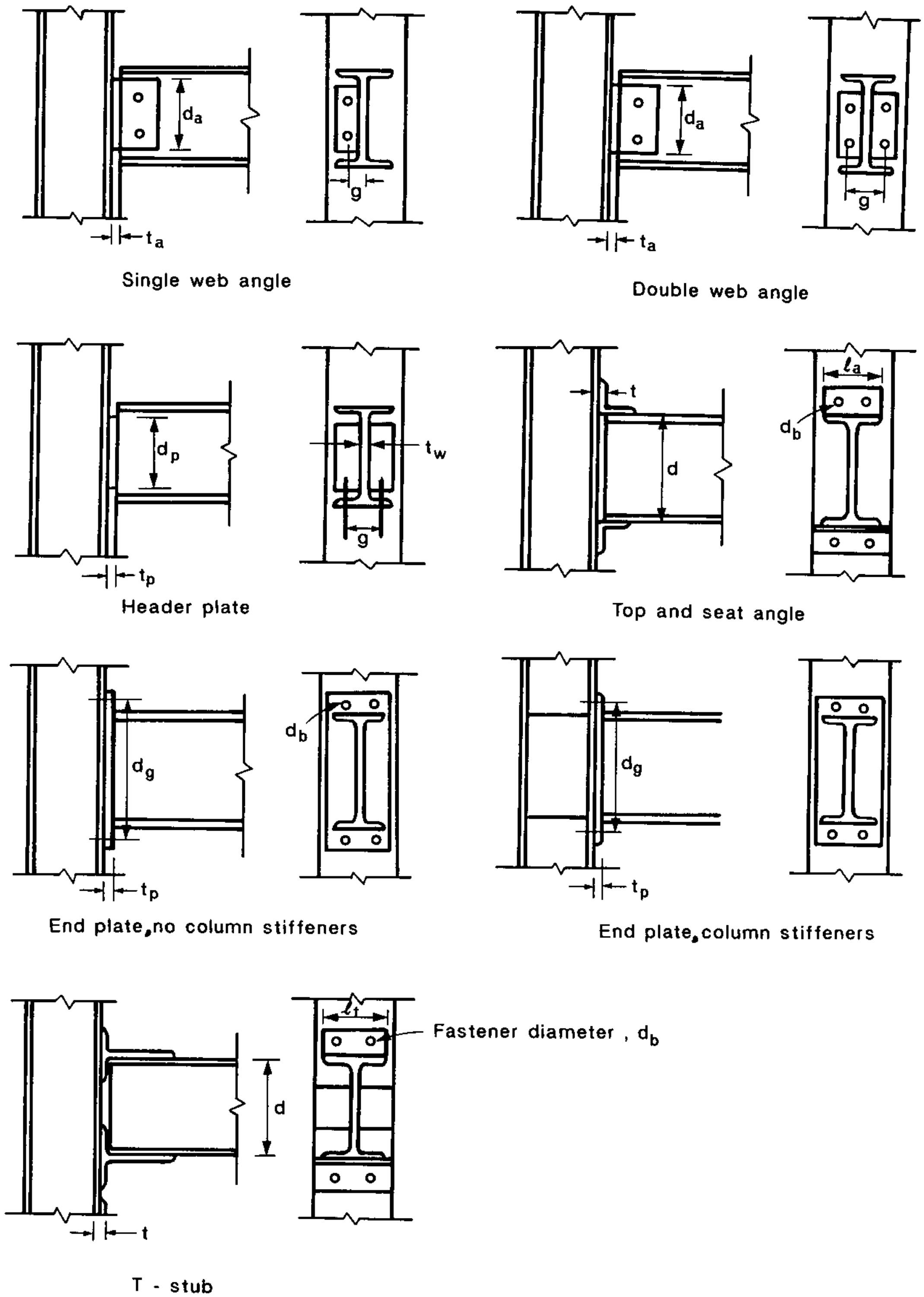
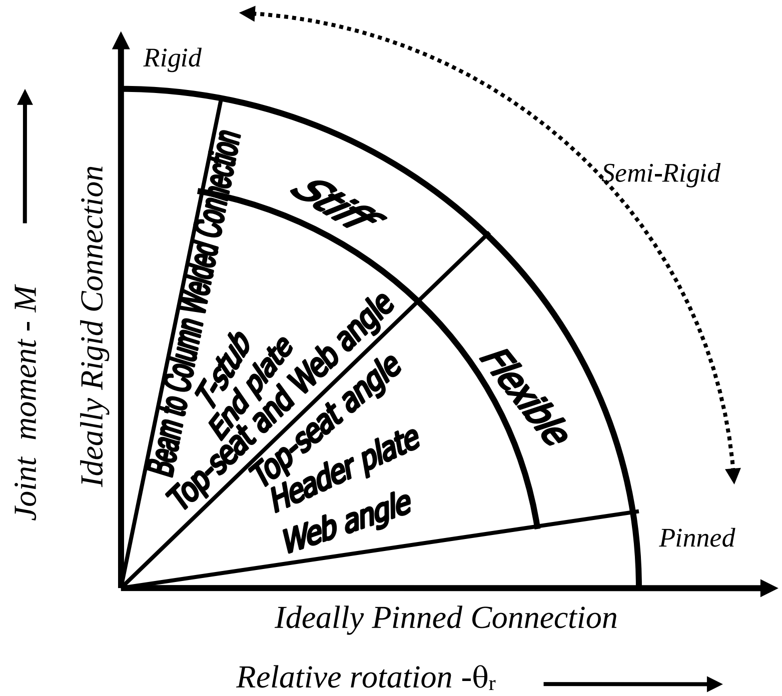


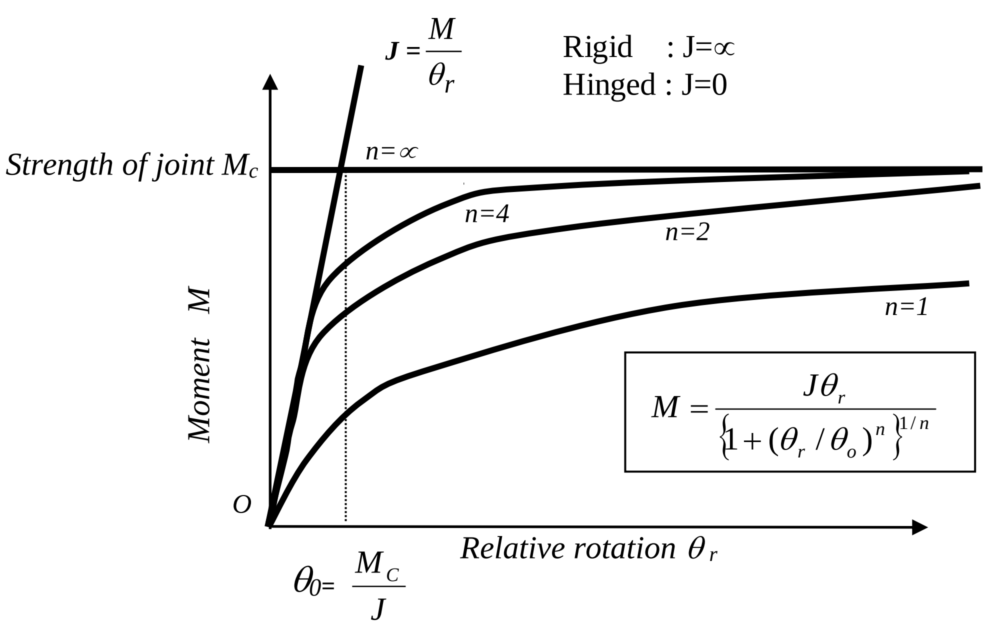

***Fig.4 Some typical structural steel connections***
***(Chen W.F and Lui E.M., "Stability Design of Steel Frames", CRC Press, 1991***

They can be broadly classified as:

- B-spline models
- Polynomial models
- Exponential models and
- Power models.




*(a) Classification of structural steel connections according to their stiffness*




*(b) Three parameter model for M-$\theta_r$ relationship*

***Fig.5 Connection Stiffness and their representation***

One such popular model is the Kishi and Chen (1990) three-parameter power model as shown in Fig.5 (b). The experimental data are fitted into a curve using three parameters such as *J*, $M_C$ and *n*. By suitably adjusting the value of $M_c$ and *'n'*, a family of $M$-$\theta_r$ curves could be generated. However, for the subsequent part of our discussion we are interested in the linear connection behaviour, and hence only the connection modulus '*J*' alone is of interest to us. It is to be noted that in the case of nonlinear analysis, all the parameters are needed.

## 4.0 DERIVATION OF BASIC EQUATIONS FOR THE ANALYSIS OF SEMI-RIGID STEEL FRAMES USING MOMENT DISTRIBUTION METHOD

In this section, we would see as to how semi-rigid or flexibly connected steel frames could be analysed using the popular "Moment Distribution Method (MDM)".Since MDM has been well documented in engineering text books, the fundamentals of MDM would not be repeated here. The following discussions are based on the assumption that the reader has prior knowledge of MDM.

As we have seen earlier, semi-rigid steel frames could be idealised as bare steel frames with connections modelled as flexural springs as in Fig.1(b). Hence, it is apparent that to model the connection flexibility using MDM, the first step in the analysis is the determination of moment distribution factors for a beam (which are based on connection stiffnesses) with a spring at one end or springs at both ends. Firstly we would see individual members having flexible connections at one or both ends and later we would consider the entire steel frame to be composed of these individual members. When there is a flexible connection at each end of the beam, the stiffness and carry-over factors can be derived from a consideration of the beam shown in Fig.6.

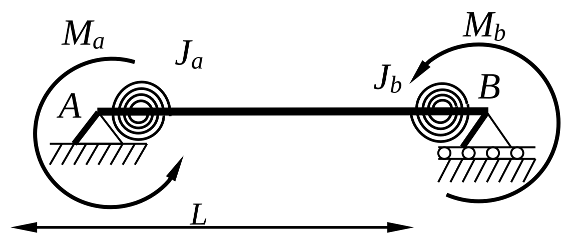



***Fig.6 Beam with connection springs at both ends***

The beam is simply supported at the ends and the joint modulii are $J_a$ and $J_b$ at ends *A* and *B*, respectively. Under the action of moments $M_a$ and $M_b$ acting at the ends, the ends of the beam will rotate through small angles $\theta_a$ and $\theta_b$, which are assumed positive in the same directions as the positive end moments as shown in Fig 6. The angles of rotation at the ends of the portion of the beam between the springs could be written as

$$\frac{M_aL}{3EI}-\frac{M_bL}{6EI} \text{ and } -\frac{M_aL}{6EI}+\frac{M_bL}{3EI} \tag{1}$$

Due to rotations of the elastic springs representing the connections, the ends of the beam rotate through additional angles equal to

$$\frac{M_a}{J_a} \text{ and } \frac{M_b}{J_b} \tag{2}$$

Hence the total angles of rotation of the ends of the beam in Fig.6 is given by

$$\theta_a = \frac{M_a L}{3EI} - \frac{M_b L}{6EI} + \frac{M_a}{J_a} \quad \text{(a)}$$

$$\theta_b = -\frac{M_a L}{6EI} + \frac{M_b L}{3EI} + \frac{M_b}{J_b} \quad \text{(b)} \tag{3}$$

The above equations are fundamental in nature using which we can calculate the stiffness and carry-over factors of any member with flexible connection at it ends.

**4.1 Members with far end fixed and flexible connections at both ends**

If the far end of the beam *AB* is fixed (Fig.7), the angle of rotation at end *B* is zero. The carry-over factor from end *A* to the end *B* can be found by solving Eq.3.(b) for the ratio of $M_b$ to $M_a$; we get

$$COF_{ab} = \frac{1}{2\left(1 + \frac{3EI}{LJ_b}\right)} \tag{4}$$

Introducing a factor called *'j'*

$$j = \frac{EI}{LJ} \tag{5}$$

where '*j*' is a dimensionless quantity called the *joint factor*. Hence Eq.4 becomes

$$COF_{ab} = \frac{1}{2(1+3j_b)} \tag{6}$$

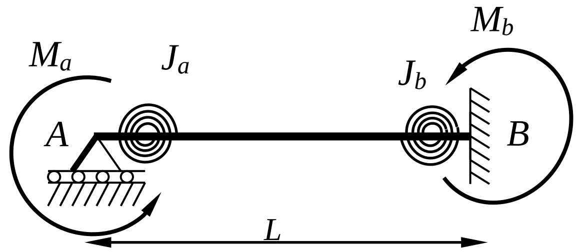



***Fig.7 Beam with far end fixed***

On the other hand, if the connection at the far end of the beam is rigid instead of flexible, it represents a rigid connection and it is equivalent to a joint with an infinitely large joint modulus *J*. The corresponding value of the joint factor '*j*' is zero (see Eq.5) and when this value is substituted into Eq.6 the carry-over factor becomes 0.5, a well known factor for the carry over moment in case far end is fixed.

### 4.2 Members with far end pinned and flexible connections at both ends

Similarly, if the connection at the far end *B* is completely flexible and offers no restraint against rotation, the value of *J* is zero, the joint factor *j* becomes infinite, and Eq.6 gives a carry-over factor of zero which corresponds to a beam with the far end pinned. In such a case, the rotational stiffness of the beam is obtained from Eqs.3 (a) by substituting for $M_b$ its expression in terms of $M_a$ and solving for the ratio $M_a/\theta_a$ which is the rotational stiffness. This manipulation gives the fundamental formula

$$K_{ab} = \frac{4EI}{L}\frac{1+3j_b}{1+4(j_a+3j_aj_b+j_b)} \tag{7}$$

### 4.3 Members with extremes of joint stiffness

In special cases such as connections at both the ends are rigid *( i.e.* $j_a=j_b=0$*), Eq.7* reduces to the well known results of $K_{ab}=4EI/L$ for a beam with the far end fixed. When the connections at *A* is rigid ($j_a=0$) and the connection at *B* offers no restraint against rotation ($j_b=\infty$) the result is $K_{ab}=3EI/L$, again a standard stiffness value for a beam with far end pinned.

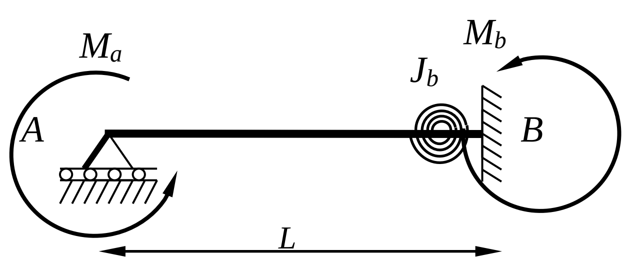



***Fig.8 Beam with flexible connection only at the far end***

When the connections at the ends of the beam are identical ($j_a=j_b=j$) the stiffness of the beam is

$$K_{ab} = \frac{4EI}{L}\frac{1+3j}{(1+2j)(1+6j)} \tag{8}$$

### 4.4 Beam with flexible connection at one end and far end fixed

Sometimes it is quiet common to have a flexible connection only at one end of the member. The case, as shown in Fig.8, has a flexible connection at the far end only.

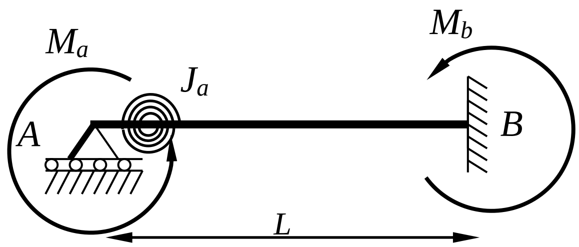



***Fig.9 Beam with flexible connection at near end and far end fixed***

This can be considered as a special case of the beam in which the spring at the near end A of the member has an infinitely large joint modulus ($j_a=\infty$). Thus, by making use of Eq.6 and Eq.7, the carry-over and stiffness factors can be written as

$$COF_{ab} = \frac{1}{2(1+3j_b)} \tag{9}$$

$$K_{ab} = \frac{4EI}{L}\frac{1+3j_b}{1+4j_b} \tag{10}$$

The case in which the flexible connection is located at the near end of the member and the far end is fixed (shown in Fig.9) can be obtained from Eq.6 and Eq.7 by substituting $j_b=0$.

Thus the carry-over and stiffness factors for this case become

$$COF_{ab} = 1/2 \tag{11}$$

$$K_{ab} = \frac{4EI}{L}\frac{1}{1+4j_a} \tag{12}$$

**4.5 Beam with flexible connection at near end and far end pinned**

When the far end of the beam is simply supported instead of fixed (Fig.10) the carry-over factor is zero and the stiffness factor is

$$K_{ab} = \frac{3EI}{L}\frac{1}{1+3j_a} \qquad 13)$$

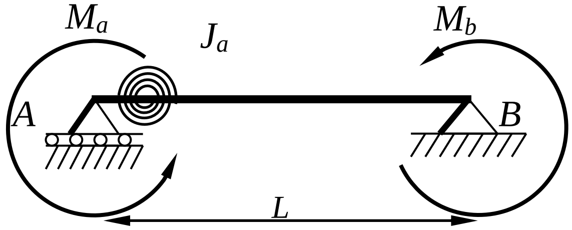



***Fig.10 Beam with flexible connection at near end and far end pinned***

### 4.6 Fixed end moments for beams with flexible connections

In the earlier sections we discussed the two important parameters for MDM, namely the stiffness and carry over factors. Another important parameter for the MDM is the 'fixed end moments'. A fixed – end beam carrying a uniform load of intensity 'w' is shown in Fig.11 It is assumed that the flexible connections at the ends of the beam are identical and have a joint modulus equal to *J*.

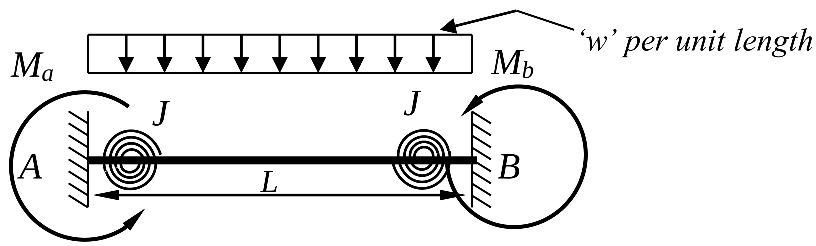



***Fig.11 Beam with UDL and flexible connections at both ends***

Hence the beam is symmetrical and the fixed – end moments are numerically equal but opposite in sign ( $M_b = -M_a$ ). By suitable manipulations it could be shown that the fixed end moments are given by

$$M_a = -M_b = \frac{wL^2}{12}\frac{1}{1+2j} \tag{14}$$

The above equations for fixed end moments are derived based on the assumption that the connection modulus $J_a = J_b = J$. But in actual practice this need not be the case. Hence considering any end of the beam to be flexible ( End '*B*' is assumed to be flexible in Eq.15&16) we can show the fixed end moments to be

$$M_a = \frac{wL^2}{12}\frac{(1+6j_b)}{(1+4j_b)} \tag{15}$$

$$M_b = \frac{wL^2}{12}\frac{-1}{1+4j_b} \tag{16}$$

For a case where $J_a \neq J_b$ , the fixed end moments could be obtained by simple algebraic addition using eqn.15&16 by suitably substituting $J_a$ and $J_b$ values. The fixed end moment caused by a concentrated load P acting at a distance '*kl'* from the left end of the beam (Fig.12) can be shown to be

$$M_a = PLk(1-k)\frac{1+4j-k(1+2j)}{(1+2j)(1+6j)} \tag{17}$$

$$M_b = -PLk(1-k)\frac{2j+k(1+2j)}{(1+2j)(1+6j)} \tag{18}$$

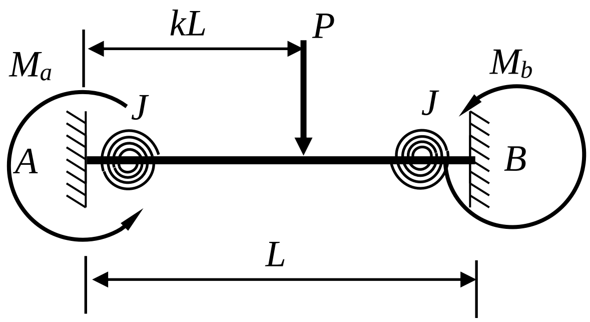



***Fig.12 Beam with Concentrated load and flexible connections at both ends***

When the connections are rigid (*j=0*) these expressions reduce to the usual formulas for fixed end moments. As before, considering the joint modulii at the ends of the member different, we get the fixed end moments for a flexibly connected beam under concentrated load as,

$$M_a = \frac{PkL(1-k)}{2(1+3j_a)}\left((2-k) - \frac{(2(1+k)(1+3j_a)-(2-k))}{3(1+4j_b)(1+3j_a)}\right) \quad (19)$$

$$M_b = -\frac{PkL(1-k)}{3(1+4j_b)(1+3j_a)}(2(1+k)(1+3j_a)-(2-k)) \quad (20)$$

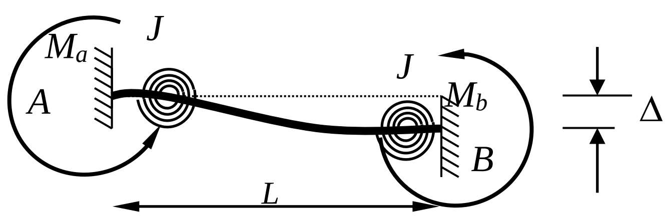



***Fig.13 Member with sway deflection***

## 4.7 Joint Translation in sway frames

In the case of sway frames, the member ends also experience lateral displacement. Fixed end moment formulae for beams in which one end is displaced laterally with respect to the other can be obtained without difficulty. For example, if both ends of the beam are fixed as in Fig.13, the fixed end moments could be shown to be

$$M_a = M_b = \frac{6EI\Delta}{L^2}\frac{1}{1+6j} \quad (21)$$

which reduce to $M_a = M_b = \dfrac{6EI\Delta}{L^2}$ when the connections are rigid (*j=0*). If there is flexibility at only one end (Fig.14)of the beam, the stiffness and carry–over factors are not the same at each end of the beam; but must be obtained from separate expressions. The carry – over and the stiffness factors at end A are

$$COF_{ab} = \frac{1}{2} \qquad K_{ab} = \frac{4EI}{L}\frac{1}{1+4j} \tag{22}$$

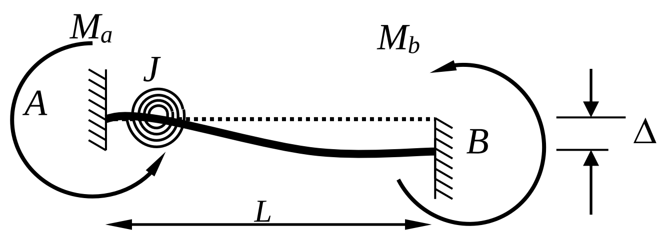



***Fig.14 Member with sway deflection and flexible connection at one end***

The corresponding quantities at end B are

$$COF_{ab} = \frac{1}{2(1+3j)} \qquad K_{ab} = \frac{4EI}{L}\frac{1+3j}{1+4j} \tag{23}$$

Using the above four expressions the fixed – end moments could be calculated as

$$M_a = \frac{6EI\Delta}{L^2}\frac{1}{1+4j} \qquad M_b = \frac{6EI\Delta}{L^2}\frac{1+2j}{1+4j} \tag{24}$$

For a case where $J_a \neq J_b$ , the fixed end moments could be obtained by simple algebraic addition using eqn.24. As a final case, it is assumed that there is a flexible connection at one end of the beam and that the other end of the beam is simply supported (Fig.15). The moment $M_a$ at the fixed end of the beam is equal to the moment which is required to rotate that end of the beam through an angle $\Delta / l$. This moment is equal to the stiffness factor for the beam with the far end simply supported times $\Delta / l$; therefore

$$M_a = \frac{3EI\Delta}{L^2}\frac{1}{1+3j_a} \tag{25}$$

To summarise the above sections, it was demonstrated as to how the important parameters such as stiffness, carryover factor and fixed end moments could be derived from principles of mechanics. Once these basic expressions are available, the entire semi-rigid multi-storey steel frame could be considered as made up of these basic components.

## 5.0 ANALYSIS OF SEMI-RIGID STEEL FRAMES

Using the expressions presented above we would solve some problems to understand the analysis of frames with semi-rigid connections. Let us take an example of a continuous beam as shown in Fig.16.

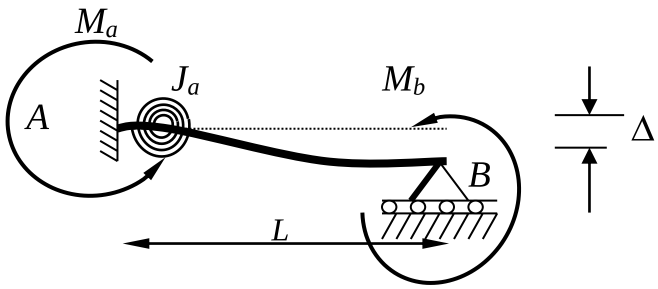



*Fig.15 Member with sway deflection with far end pinned*

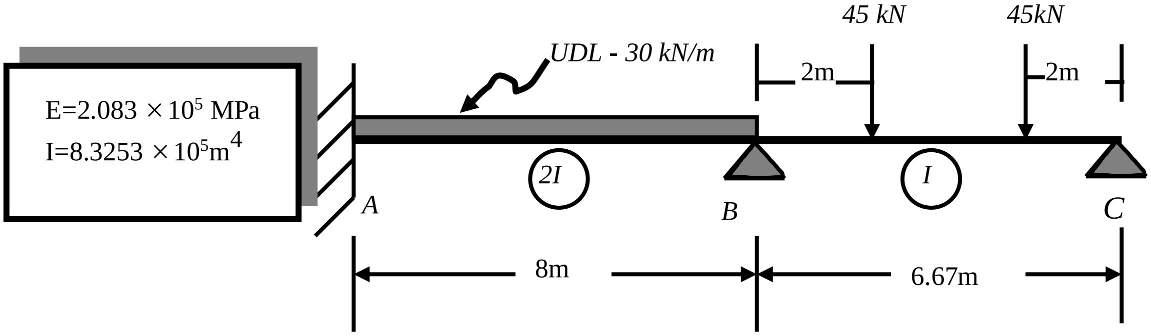



*Fig.16 Continuous beam with rigid connections*

***Table 1***

| | ***End*** | *AB* | *BA* | *BC* | *CB* |
|---|---|---|---|---|---|
| | ***DF*** | | 0.625 | 0.375 | |
| | ***COF*** | | 0.5 | 0.5 | 0.5 |
| | ***FEM*** | +160.000 | -160.000 | +63.014 | -63.014 |
| ***Iteration 1*** | ***Balance B*** | | +60.616 | +36.370 | |
| | ***Carry over*** | 30.308 | | | +18.185 |
| | ***Balance C*** | | | | +44.829 |
| | ***Carry over*** | | | 22.415 | |
| ***Iteration 2*** | ***Balance B*** | | -14.010 | -8.406 | |
| | ***Carry over*** | -7.005 | | | -4.203 |
| | ***Balance C*** | | | | +4.203 |
| | ***Carry over*** | | | +2.102 | |
| ***Iteration 3*** | ***Balance B*** | | -1.314 | -0.788 | |
| | ***Carry over*** | -0.657 | | | -0.394 |
| | ***Balance C*** | | | | +0.394 |
| | ***Final moments (app.)*** | **+182.646** | **-114.708** | **+114.708** | **0.000** |

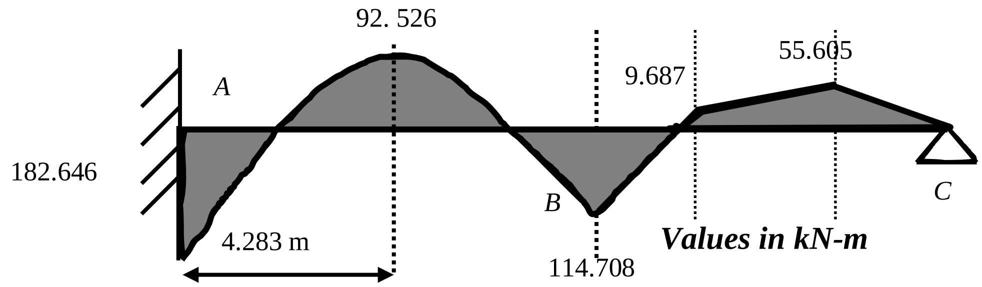



***Fig.17 Bending Moment Diagram (rigid case)***

At the first instance, let us assume that the support at A is rigid and accordingly we would work out the stiffness of joints, distribution factors and carry over factors. We obtain distribution factors $D_{BA} = 0.625$ and $D_{BC} = 0.375$ based on stiffnesses $K_{BA}, K_{BC}$. Since the connections are assumed perfectly rigid, half the moment induced at *B* and *C* would be carried over to the adjuscent joint. Regarding the hinged node *C*, there are two ways to handle.

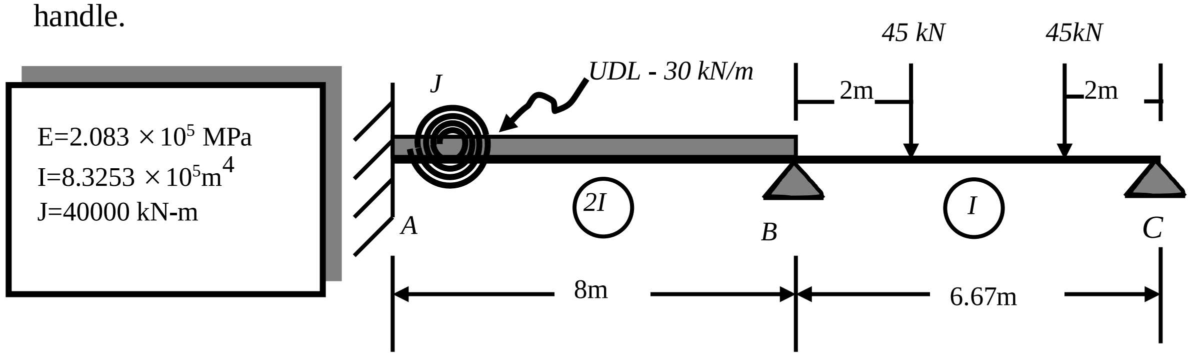



***Fig.18 Continuous beam with flexible connection***

***Table 2***

| | ***End*** | *AB* | *BA* | *BC* | *CB* |
|---|---|---|---|---|---|
| | ***DF*** | | 0.606 | 0.394 | |
| | ***COF*** | | 0.377 | 0.5 | 0.5 |
| | ***FEM*** | +111.421 | -184.290 | +63.014 | -63.014 |
| ***Iteration 1*** | ***Balance B*** | | +73.493 | +47.783 | |
| | ***Carry over*** | +27.707 | | | +23.892 |
| | ***Balance C*** | | | | +39.123 |
| | ***Carry over*** | | | +19.562 | |
| ***Iteration 2*** | ***Balance B*** | | -11.855 | -7.707 | |
| | ***Carry over*** | -4.469 | | | -3.854 |
| | ***Balance C*** | | | | +3.854 |
| | ***Carry over*** | | | +1.927 | |
| ***Iteration 3*** | ***Balance B*** | | -1.168 | -0.759 | |
| | ***Carry over*** | -0.440 | | | -0.380 |
| | ***Balance C*** | | | | +0.380 |
| | ***Final moments (app.)*** | **+134.219** | **-123.820** | **+123.820** | **0.000** |

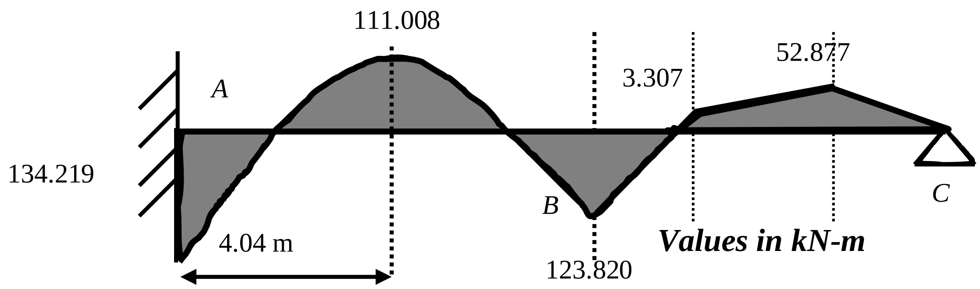



***Fig.19 Bending Moment Diagram (flexible case)***

Firstly we can get the stiffness $K_{BC}$ considering the far end *C* is hinged and obtain $K_{BC}=3EI/L$ and fixed end moment $M_{CB}$ is set to zero. Alternatively *C* could be considered as rigid, and subsequently we can balance *C* to zero and carry over the moments to *B*. The later method is adopted in the present example. The MDM is presented in Table 1 for the rigid case. To start with, all the nodes are assumed to be locked. First we unlock node *B*. An unbalanced moment of –96.986 kN-m appears which is balanced by distributing it at node *B*. Now because of the appearance of the new balancing moments half the moment is carried over to adjacent end. This is done in the carry over column as shown in Table 1. This introduces unbalancing moments at node *C*, which is then balanced and moments carried over. Now we have completed one cycle. Similarly we can repeat this exercise until two consequent change in moment at any node is within an acceptably small value. However in the present example only three iterations are shown in Table 1. We see from the results (Fig.17)that the ratio of negative support moment at *A* to the positive span moment in *AB* is 1.97. We shall consider the same example but assume that the connection at *A* is flexible and the connection stiffness *J*=40000 kN-m/rad. The problem is shown in Fig.18 and the procedure is presented in Table 2. From Table 2 we observe that the connection flexiblity affects several parameters. Firstly the stiffness of a particular joint gets reduced if the far end connection is flexible. We see that the stiffness $K_{BA}$ is reduced and hence gets only 0.606 time the connection moment as against 0.625 in the rigid case. The Remainder of the moment is distributed to the other members connected to it. Similarly we also see from Table 2, that the moment carried over to the far end gets reduced because of the connection flexiblity. Another important observation is that the fixed end moment is reduced at the end where the connection flexiblity occurs leaving a increased share of the end moment to the rigid end. Hence we see(Fig.19) that fixed end moment $M_{AB}$ is reduced to 134.219 kN-m from 186.646kN-m in the rigid case. At the same time $M_{BA}$ increased from 114.708 kN-m to 123.820 kN-m. The final end moments are presented in Table 2.

We see that the , in the span *AB*, ratio of negative moment at *A* to the maximum positive span moment is brough down to 1.209. The design bending moment in the span *AB* has reduced by 36%. The reduced design moment is one of the main advantages of semi-rigid steel frames. In the chapter on “Welds- Static and Fatigue Strength –II’, the effect on connection flexiblity on the moment redistribution is well explained. Since steel beams are equally good both in compression and tension, we see that there is a better utilisation of the material of the beam for carrying the load in flexure. If we see the chapter on 'Plastic Analysis', this is exactly what we are trying to achieve. In an ideal situation we could get a ratio of the negative bending moment to positive span moment as 1.0.

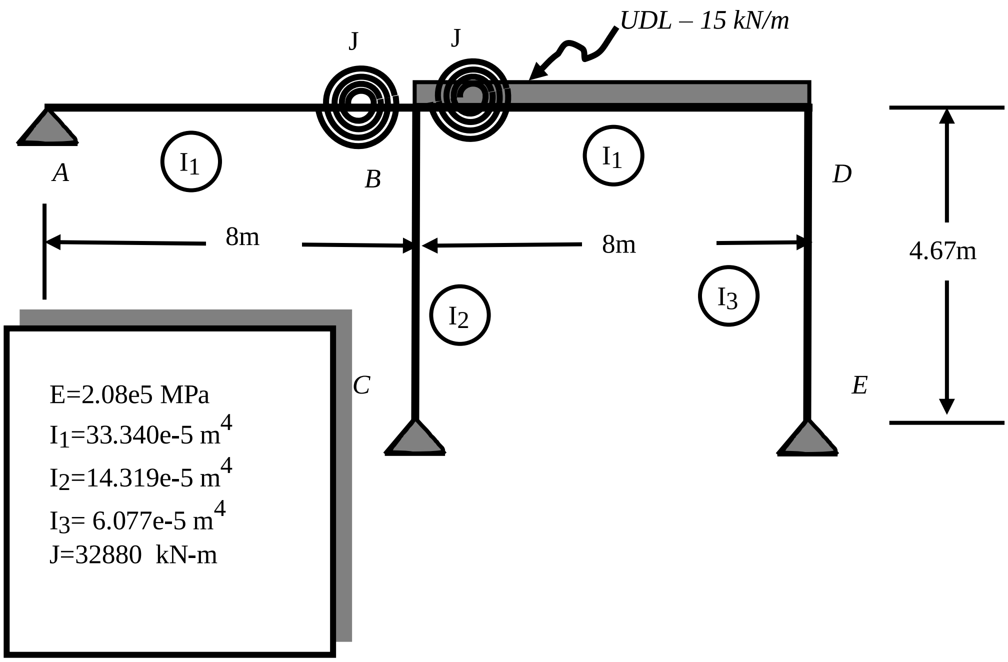



***Fig.20 Single storey steel frame***

***Table 3***

| ***Rigid Connection*** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| End | AB | BA | BC | BD | CB | DB | DE | ED |
| DF | | 0.366 | 0.269 | 0.366 | | 0.762 | 0.238 | |
| COF | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |
| FEM | 0.0 | 0.0 | 0.0 | 80.0 | 0.0 | -80.0 | 0.0 | 0.0 |
| Final end Moments | 0.0 | -40.16 | -29.55 | 69.71 | 0.0 | -20.21 | 20.21 | 0.0 |
| ***Flexible connection*** | | | | | | | | |
| End | AB | BA | BC | BD | CB | DB | DE | ED |
| DF | | 0.284 | 0.432 | 0.284 | | 0.762 | 0.238 | |
| COF | 0.278 | 0.500 | 0.500 | 0.500 | 0.500 | 0.278 | 0.500 | 0.500 |
| FEM | 0.0 | 0.0 | 0.0 | 38.740 | 0.0 | -100.6 | 0.0 | 0.0 |
| Final end Moments | 0.0 | -18.11 | -23.96 | 42.07 | 0.0 | -23.55 | 23.55 | 0.0 |

Another example of a single storey frame is provided as an illustration as shown in Fig.20. The distribution and carryover factors for the rigid and semi-rigid case are presented in Table 3. One can observe the change in the fixed end moments in Table.3. The problem could be solved manually or by using the program presented in the Appendix. From the final end moments it is observed that the maximum moment has been brought down to 42.07 kN-m from 69.71 kN-m. We also observe that connection flexibility results in redistribution of moments and a better utilisation of the beam material.

The procedure explained above could be extended to any multi storey steel frame. However as more number of storeys and bays are considered, the hand computation of MDM becomes very laborious. Nevertheless, the MDM could be programmed as computer software. The ideal solution for semi-rigid analysis of steel frames is the Finite Element Method (FEM) as it provides greater flexibility in modelling. Since treatment of FEM is outside the scope of this chapter, it will suffice to know that FEM could be used very effectively for both linear and non-linear analysis of semi-rigid steel frames.

## 6.0 SEMI-RIGID DESIGN OF FRAMES

Many of the codes of practice allow the use of semi-rigid design methods for steel frames. IS:800(1984) also allows the semi-rigid design methods provided some rational analysis procedures are used. However the code does not elaborate any further. For example, BS:5950 Part –1 allows semi-rigid design stating that (Clause 2.12.4) " The moment and rotation capacity of the joint should be based on experimental evidence which may permit some limited plasticity provided that the ultimate tensile capacity of the fasteners is not the failure criterion". Euro Code (EC3) also allows the semi-rigid design methods and the main provisions could be summarised as follows:

- Moment –rotation behaviour shall be based on theory supported by experiments.
- The real behaviour may be represented by a rotational spring.
- The actual behaviour is generally nonlinear. However, an appropriate design curve may be derived from a more precise model by adapting linear approximations such that the whole curve lies below the accurate curve as shown in Fig.21.
- Three properties are defined in the $M$-$\theta_r$ characteristics
    - Maximum moment of resistance ($M_C$ )
    - Rotational stiffness (the secant stiffness $J=M/\theta_r$
    - The rotation capacity $\theta_c$

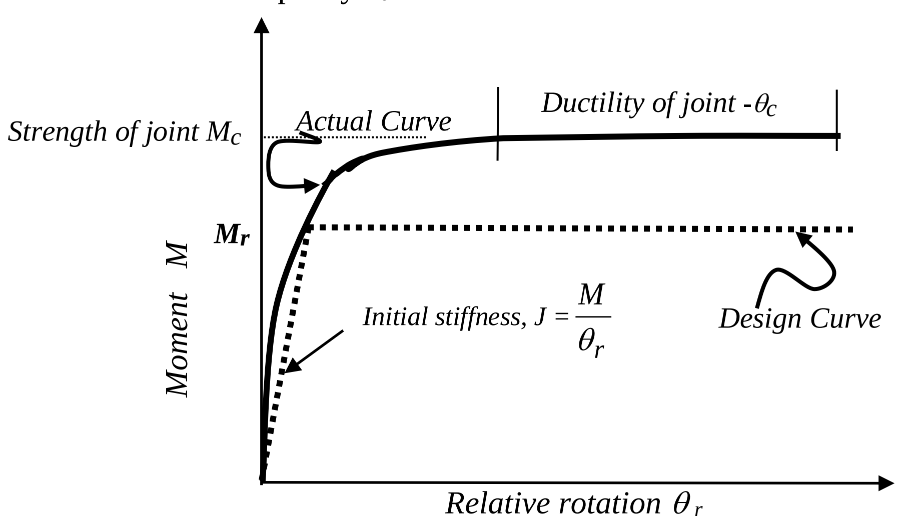



***Fig.21 Typical Design curve for semi-rigid joints***

In the design of components such as beams, columns and beam columns the procedure is the same as in the rigid elastic design of multi storey frames. Only in the case of columns and beam columns, the effective lengths of members have to be ascertained using alignment charts which considers connection flexibility or by an elaborate instability analysis.

## 7.0 COMPUTER PROGRAM "FLEXIFRAME" FOR THE SEMI-RIGID ANALYSIS OF STEEL FRAMES

A FORTRAN computer program "FLEXIFRAME" has been written to incorporate the derived flexibility equations using Moment Distribution Method (MDM) derived in this chapter. The computer implementation of the MDM results in the Gauss – Seidel iteration method. The program is capable of analysing non-sway steel frames with flexible connections. However with little modifications, the program could be extended to the analysis of sway frames. The computer program FLEXIFRAME has been presented in Appendix. The input details of the program have also been given in Appendix. The reader is encouraged to try out various problems of multi-storey semi-rigid steel frames to understand the effect of connection flexibility using the computer program.

## 8.0 SUMMARY

In this chapter, the fundamentals of connection flexibility in steel frames are described. The stiffness equations for the semi-rigid analysis of steel frames using the popular moment distribution method are derived. Example problems, which use the derived stiffness equations, have been presented. The fundamental differences between the behaviour of fully rigid and semi-rigid frames have been brought out. The importance of experimental evaluation of the connection stiffness has also been described. Finally a brief outline of the design procedures has been presented.

## 9.0 REFERENCES


1. Gere J.M., "Moment distribution", D. Van Nostrand Co. Inc, NY, (1963)
2. Chen W.F. and Lui E.M., "Stability design of steel frames", CRC Press Inc., (1991).
3. Cornelius T. " Techniques in buildings and bridges", Gordon and Breach Int. series, Vol.11,(1999)
4. Kishi N. and Chen W.F.," Moment –rotation relations of semi-rigid connections with angles", Journal of Structural Engineering, ASCE, 116(7), 1813-1834, (1990).

# ***Appendix***

```
c    A computer program to analyse non-sway semi-rigid steel frames
     Program FLEXIFRAME
     parameter (nsize=50)
     character *12 inpf,outf
     character *87 tit
     real xlen(50),mi(nsize),jm(nsize,2),jstiffa,jstiffb,ja,jb,kval
     integer cvity(nsize,5)
     common/loads/udlval(100),nconc,p(10),a(10),b(10)
     dimension nconnect(nsize),ie(nsize,2),distf(nsize,5)
     dimension var(nsize,nsize),cof(nsize,nsize),fem(nsize,nsize)
     dimension fimom(nsize,nsize),ibc(nsize,2),stiff(nsize,nsize)
     write(*,*)'enter input file name'
     read(*,'(a\)')inpf
     write(*,*)'enter output file name'
     read(*,'(a\)')outf
     open(10,file=inpf)
     open(11,file=outf)
c     title
     read(10,'(a)')tit
          write(11,'(a)')tit
c     general data
     read(10,*)nmem,nnode,ymod,niter
     write(11,*)nmem,nnode,ymod,niter
c     nodal data
     do 10 i=1,nnode
        read(10,*)m,nconnect(m)
        write(11,*)m,nconnect(m)
        read(10,*)(cvity(m,j),j=1,nconnect(i))
        write(11,*)(cvity(m,j),j=1,nconnect(i))
10    continue
c     memeber data
     do 20 i=1,nmem
        read(10,*)m,xlen(m),mi(m),ie(m,1),ie(m,2),jm(m,1),jm(m,2)
    .          ,ibc(m,1),ibc(m,2)
        write(11,*)m,xlen(m),mi(m),ie(m,1),ie(m,2),jm(m,1),jm(m,2)
    .          ,ibc(m,1),ibc(m,2)
        n1=ie(m,1)
        n2=ie(m,2)
c        initialise the fixed end moments
        fem(n1,n2)=0.0
       fem(n2,n1)=0.0
        jstiffa=jm(m,1)
        jstiffb=jm(m,2)
        ja= (ymod*mi(m)) / (xlen(m)*jstiffa)
        jb= (ymod*mi(m)) / (xlen(m)*jstiffb)
c       to determine the stiffness values
        st=(4.*ymod*mi(m)) / xlen(m)
        xnum=1. + 3.* jb
        den =1. + 4.*(ja+3.*ja*jb + jb)
        stiff(n1,n2)=st*(xnum/den)
        xnum=1. + 3.* ja
        den =1. + 4.*(ja+3.*ja*jb + jb)
        stiff(n2,n1)=st*(xnum/den)
```

```
c       carry over factor
        cof(n1,n2)=0.5 * ( 1./ (1.+ 3.*jb))
        cof(n2,n1)=0.5 * ( 1./ (1.+ 3.*ja))
c       load data
        read(10,*)udlval(i),nconc
        write(11,*)udlval(i),nconc
        fixm=(udlval(i)*xlen(m)*xlen(m)) / 12.
       denudl=(3.+12.*ja+12.*jb+36.*ja*jb)
        fem(n1,n2)=         fixm * ((3.*(1.+6.*jb))/ denudl)
        fem(n2,n1)= (-1.0)* fixm * ((3.*(1.+6.*ja))/ denudl)
        do 30 j=1,nconc
          read(10,*)p(j),a(j),b(j)
          write(11,*)p(j),a(j),b(j)
          kval=a(j)/xlen(m)
          ylen=xlen(m)
                pval=p(j)
          call femconc(kval,ylen,ja,jb,pval,fema,femb)
          fem(n1,n2)=fem(n1,n2) + fema
          fem(n2,n1)=fem(n2,n1) + femb
30       continue
20    continue

c     compute distribution facors
     do 40 k=1,nmem
       n1=ie(k,1)
       n2=ie(k,2)
c       for 'i' node
c       sum stiffness of members meeting at 'i' node
       stsum=0.0
       do 60 j=1,nconnect(n1)
         stsum=stsum + stiff(n1,cvity(n1,j))
60       continue
       distf(n1,n2)=(-1.0)*(cof(n1,n2)*stiff(n1,n2) / stsum)
c       for 'j' node
c       sum stiffness of members meeting at a point
         stsum=0.0
         do 80 j=1,nconnect(n2)
           stsum=stsum + stiff(n2,cvity(n2,j))
80         continue
         distf(n2,n1)=(-1.0)*(cof(n2,n1)*stiff(n2,n1) / stsum)
40    continue
c     initialise var
     do 81 i=1,nmem
       n1=ie(i,1)
       n2=ie(i,2)
       var(n1,n2)=0.0
       var(n2,n1)=0.0
81    continue
c     the main Gauss - Seidel iterartion starts here
     do 90 i=1,niter
       do 100 j=1,nmem
         n1=ie(j,1)
         n2=ie(j,2)
         if(ibc(j,1) .ne. 1)then
c            sum of the fixed end moments at 'i' node
           m1=nconnect(n1)
```

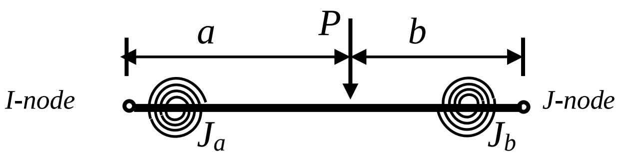
```
            sumfix=0.0
            do 110 k=1,m1
               sumfix=sumfix + fem(n1,cvity(n1,k))
110            continue
c             to find the sum of 'var' meeting at 'i' node
            sumvar=0.0
            do 120 k=1,nconnect(n1)
               sumvar=sumvar + var(cvity(n1,k),n1)
120            continue
            var(n1,n2)=distf(n1,n2) * (sumfix + sumvar)
          endif
          if(ibc(j,2) .ne. 1)then
c             sum of the fixed end moments at 'j' node
            m1=nconnect(n2)
            sumfix=0.0
            do 111 k=1,m1
               sumfix=sumfix + fem(n2,cvity(n2,k))
111            continue
c             to find the sum of 'var' meeting at 'j' node
            sumvar=0.0
            do 121 k=1,nconnect(n2)
                sumvar=sumvar + var(cvity(n2,k),n2)
121            continue
            var(n2,n1)=distf(n2,n1) * (sumfix + sumvar)
          endif
100       continue
90      continue

c     computation of final moments
      write(*,*)'***** Final support moments *****'
             write(11,*)'***** Final support moments *****'
      write(*,*)'Member no:    I-node moment          J-node moment'
      write(11,*)'Member no:    I-node moment          J-node moment'
      do 130 i=1,nmem
         n1=ie(i,1)
         n2=ie(i,2)
         fimom(n1,n2)=fem(n1,n2) + (var(n1,n2)/cof(n1,n2)) + var(n2,n1)
         fimom(n2,n1)=fem(n2,n1) + (var(n2,n1)/cof(n2,n1)) + var(n1,n2)
999           continue
         write(*,888)i,fimom(n1,n2),fimom(n2,n1)
         write(11,888)i,fimom(n1,n2),fimom(n2,n1)
130     continue
888     format(i3,10x,f15.4,7x,f15.4)
      stop
      end
c     ------------------------------------------------
      subroutine femconc(kval,xlen,ja,jb,pval,fema,femb)
c     ------------------------------------------------
      real kval,num,ja,jb
      xnum=pval*kval*xlen*(1.-kval)
     den=(1.+4.*ja+4.*jb+12.*ja*jb)
     num=1.+4.*jb -kval*(1.+2.*jb)
      fema=(xnum*num)/den
      xnum=(-1.0)*pval*kval*xlen*(1.- kval)
      num=2.*ja + kval*(1.+2.*ja)
      femb= xnum*num/den
```

```
return
end
```

**Input to Flexiframe**

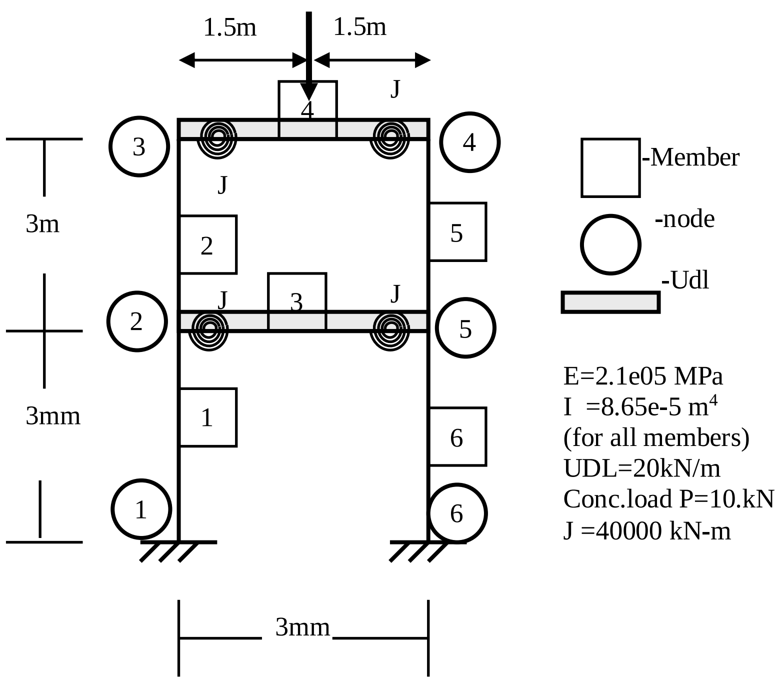
Card set No 1: nmem –number of members in the steel frame
nnode –number of nodes in the steel frame
ymod -Youngs Modulus
niter -Number of moment distribution iterations

Card set No.2: For every node
Node number, number of nodes connected to that particular node
Node numbers connected




Card set No.3: For every member
Node number, Length, Moment of inertia, I-node,J-node, Ja value, Jb value, Displacement code for I-node,Displacement code for J-node
Disp. Code -1 – joint is fixed
Disp. Code –0 - joint is pinned or it can rotate
UDL value, number of concentrated loads
For number of concentrated loads
Load value, a –distance, b-distance

Example Problem:



Input data for example Problem:
data for example problem (All units in kN -m)
6,6,2.1e05,40
1,1
2
2,3
1,3,5
3,2
2,4
4,2
3,5
5,3
2,4,6
6,1
5
1, 3.,8.65e-5,1,2,1.e20,1.e20,1,0
0. 0
2, 3.,8.65e-5,2,3,1.e20,1.e20,0,0
0. 0
3, 3.,8.65e-5,2,5,40000.,40000.,0,0
20.0  0
4, 3.,8.65e-5,3,4, 40000., 40000.,0,0
20.0 1
10. 1.5 1.5
5, 3.,8.65e-5,4,5,1.e20,1.e20,0,0
0. 0
6, 3.,8.65e-5,5,6,1.e20,1.e20,0,1
0.0 0