

<h1>Structural Steel Design Project</h1> <p>Calculation Sheet</p>	Job No:	Sheet <i>1 of 6</i>	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	<i>Worked Example - 1</i>		
		Made by <i>SSSR</i>	Date <i>24-1-2000</i>
	Checked by <i>PU</i>	Date <i>30-4-2000</i>	

Analyse the building frame shown in Fig. A using portal method.

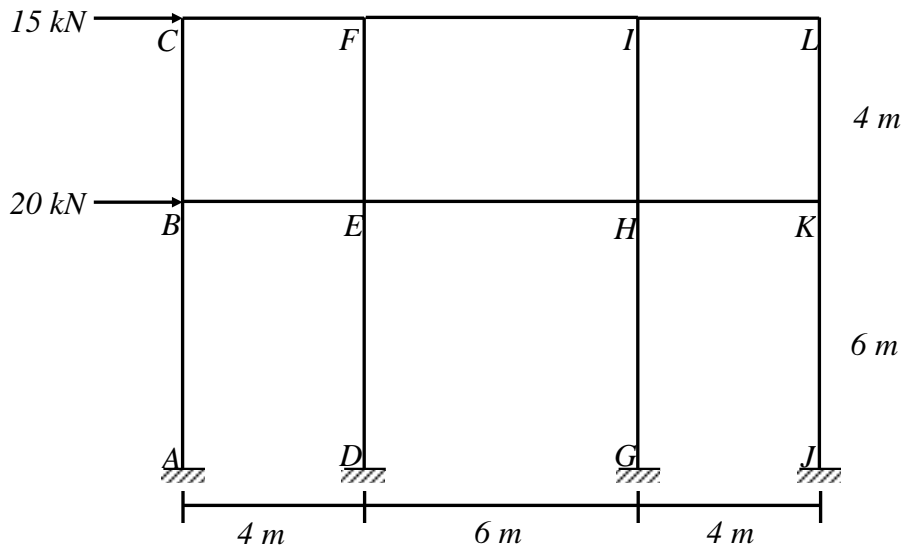


Fig. A

Top storey:

(i) **Column Shears:**

Shear in columns of the top storey is obtained by considering the free body diagram shown in Fig. A.1

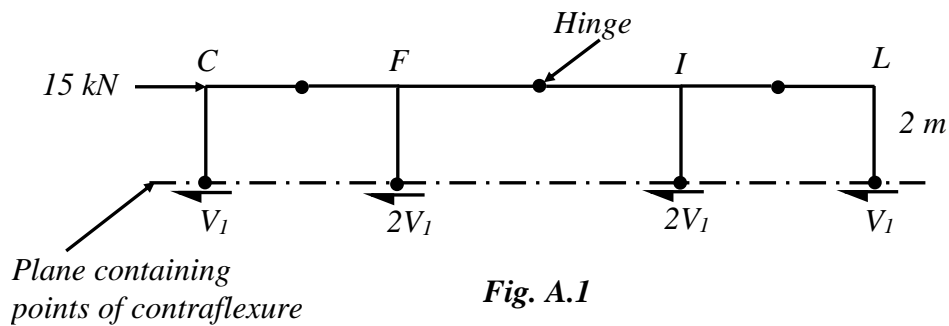


Fig. A.1

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet <i>2 of 6</i>	Rev																									
	Job Title: MULTI-STOREYED BUILDINGS																											
	<i>Worked Example – 1</i>																											
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$V_1 + 2V_1 + 2V_1 + V_1 = 15 \text{ kN} \quad [\text{Assumption 3}]$ <p>Shear in end column, $V_1 = 2.5 \text{ kN}$</p> <p>Shear in middle columns, $2V_1 = 5.0 \text{ kN}$</p> <p>Thus, shear in columns are:</p> <table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: left;"><i>Column</i></th> <th style="text-align: left;"><i>Shear (kN)</i></th> </tr> </thead> <tbody> <tr> <td><i>CB</i></td> <td>2.5</td> </tr> <tr> <td><i>FE</i></td> <td>5.0</td> </tr> <tr> <td><i>IH</i></td> <td>5.0</td> </tr> <tr> <td><i>LK</i></td> <td>2.5</td> </tr> </tbody> </table> <p>(ii) Column moments:</p> <p>Column moments are found by multiplying column shear and half the height of column as shown below:</p> <table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: left;"><i>Column</i></th> <th style="text-align: left;"><i>Shear (kN)</i></th> <th style="text-align: left;"><i>Moment (kN-m)</i></th> </tr> </thead> <tbody> <tr> <td><i>CB</i></td> <td>2.5</td> <td>$2.5 * 2 = 5.0$</td> </tr> <tr> <td><i>FE</i></td> <td>5.0</td> <td>$5.0 * 2 = 10.0$</td> </tr> <tr> <td><i>IH</i></td> <td>5.0</td> <td>$5.0 * 2 = 10.0$</td> </tr> <tr> <td><i>LK</i></td> <td>2.5</td> <td>$2.5 * 2 = 5.0$</td> </tr> </tbody> </table> <p>(iii) Girder Moments:</p> <p>At any joint, sum of the girder moments is equal to the sum of the column moments. Starting from left corner of the frame, C</p> <p>Joint C:</p> $M_{CB} = M_{CF} = 5 \text{ kN-m}$				<i>Column</i>	<i>Shear (kN)</i>	<i>CB</i>	2.5	<i>FE</i>	5.0	<i>IH</i>	5.0	<i>LK</i>	2.5	<i>Column</i>	<i>Shear (kN)</i>	<i>Moment (kN-m)</i>	<i>CB</i>	2.5	$2.5 * 2 = 5.0$	<i>FE</i>	5.0	$5.0 * 2 = 10.0$	<i>IH</i>	5.0	$5.0 * 2 = 10.0$	<i>LK</i>	2.5	$2.5 * 2 = 5.0$
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<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 3 of 6	Rev																
	Job Title: MULTI-STOREYED BUILDINGS																		
	<i>Worked Example - 1</i>																		
		Made by SSSR	Date 24-1-2000																
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<p><i>Joint F:</i></p> $M_{FC} + M_{FI} = M_{FE}$ $M_{FI} = M_{FE} - M_{FC} = 10 - 5 = 5 \text{ kN-m}$ <p><i>Joint I:</i></p> $M_{IF} + M_{IL} = M_{IH}$ $M_{IL} = M_{IH} - M_{IF} = 10 - 5 = 5.0 \text{ kN-m}$ <p>(iv) Girder shears:</p> <p><i>Girder shear = Girder Moment / (span/2)</i></p> <table border="1"> <thead> <tr> <th>Span</th> <th>Span/2 (m)</th> <th>Moment (kN-m)</th> <th>Shear (kN)</th> </tr> </thead> <tbody> <tr> <td>CF</td> <td>2.0</td> <td>5.0</td> <td>2.50</td> </tr> <tr> <td>FI</td> <td>3.0</td> <td>5.0</td> <td>1.67</td> </tr> <tr> <td>IL</td> <td>2.0</td> <td>5.0</td> <td>2.50</td> </tr> </tbody> </table> <p>(v) Column axial forces: (See Fig. A .2)</p> <p><i>Axial force on a column is determined by summing up the girder shears and other axial forces at each joint.</i></p> <p><i>Starting from the left corner of the frame, we have</i></p> <p><i>Joint C:</i></p> $\Sigma F_y = 0$ $F_{CB} = V_{CF} = 2.5 \text{ kN}$ <p><i>Joint F:</i></p> $\Sigma F_y = 0$ $F_{EF} + V_{FC} = V_{FI}$ $F_{EF} = V_{FI} - V_{FC} = 1.67 - 2.5 = -0.83 \text{ kN}$				Span	Span/2 (m)	Moment (kN-m)	Shear (kN)	CF	2.0	5.0	2.50	FI	3.0	5.0	1.67	IL	2.0	5.0	2.50
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	<i>Worked Example - 1</i>																						
	Made by SSSR		Date 24-1-2000																				
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<p><i>Joint I :</i> $\Sigma F_y = 0$ $V_{IF} = F_{IH} + V_{IL}$ $F_{IH} = V_{IF} - V_{IL} = 1.67 - 2.5 = - 0.83 \text{ kN}$</p> <p><i>Joint L :</i> $\Sigma F_y = 0$ $F_{KL} = V_{LI} = 2.5 \text{ kN}$</p> <p>Ground storey:</p> <p><i>(i) Column shears:</i></p> <p><i>Shear in the columns of ground storey is calculated in similar way to the top storey. The plane containing points of contraflexure will pass through the half the height of the ground storey. Then shear in columns is calculated as</i></p> $\Sigma F_H = 0$ $6V_1 = 35 \text{ kN.}$ $V_1 = 35/6 = 5.8 \text{ kN.}$ <p><i>Column Shears;</i></p> $V_{AB} = 5.8 \text{ kN} \quad ; \quad V_{DE} = 11.7 \text{ kN}$ $V_{JK} = 5.8 \text{ kN} \quad ; \quad V_{GH} = 11.7 \text{ kN.}$ <p><i>(ii) Column moments:</i></p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Column</th> <th style="padding: 5px;">Length/2 (m)</th> <th style="padding: 5px;">Shear (kN)</th> <th style="padding: 5px;">Moment (kN-m)</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;"><i>AB</i></td> <td style="padding: 5px;">3.0</td> <td style="padding: 5px;">5.8</td> <td style="padding: 5px;">17.5</td> </tr> <tr> <td style="padding: 5px;"><i>DE</i></td> <td style="padding: 5px;">3.0</td> <td style="padding: 5px;">11.7</td> <td style="padding: 5px;">35.0</td> </tr> <tr> <td style="padding: 5px;"><i>GH</i></td> <td style="padding: 5px;">3.0</td> <td style="padding: 5px;">11.7</td> <td style="padding: 5px;">35.0</td> </tr> <tr> <td style="padding: 5px;"><i>JK</i></td> <td style="padding: 5px;">3.0</td> <td style="padding: 5px;">5.8</td> <td style="padding: 5px;">17.5</td> </tr> </tbody> </table>				Column	Length/2 (m)	Shear (kN)	Moment (kN-m)	<i>AB</i>	3.0	5.8	17.5	<i>DE</i>	3.0	11.7	35.0	<i>GH</i>	3.0	11.7	35.0	<i>JK</i>	3.0	5.8	17.5
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<h1>Structural Steel Design Project</h1> <p>Calculation Sheet</p>	Job No:	Sheet <i>5 of 6</i>	Rev																
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<p>(iii) Girder moments:</p> <p><i>Joint B:</i></p> $M_{BE} = M_{BC} + M_{BA} = 5.0 + 17.5 = 22.5 \text{ kN-m}$ <p><i>Joint E:</i></p> $M_{EH} = M_{EF} + M_{ED} - M_{EB}$ $= 10 + 35.0 - 22.5 = 22.5 \text{ kN-m}$ <p><i>Joint H:</i></p> $M_{HK} = M_{IH} + M_{HG} - M_{HE}$ $= 10 + 35.0 - 22.5 = 22.5 \text{ kN-m}$ <p><i>Joint K:</i></p> $M_{KH} = M_{KL} + M_{JK} = 5 + 17.5 = 22.5 \text{ kN-m}$																			
<p>(iv) Girder shears:</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;"><i>Beam</i></th> <th style="text-align: left;"><i>Span/2 (m)</i></th> <th style="text-align: left;"><i>Moment (kN-m)</i></th> <th style="text-align: left;"><i>Shear (kN)</i></th> </tr> </thead> <tbody> <tr> <td><i>BE</i></td> <td><i>2.0</i></td> <td><i>22.5</i></td> <td><i>11.3</i></td> </tr> <tr> <td><i>EH</i></td> <td><i>3.0</i></td> <td><i>22.5</i></td> <td><i>7.5</i></td> </tr> <tr> <td><i>HK</i></td> <td><i>2.0</i></td> <td><i>22.5</i></td> <td><i>11.3</i></td> </tr> </tbody> </table>				<i>Beam</i>	<i>Span/2 (m)</i>	<i>Moment (kN-m)</i>	<i>Shear (kN)</i>	<i>BE</i>	<i>2.0</i>	<i>22.5</i>	<i>11.3</i>	<i>EH</i>	<i>3.0</i>	<i>22.5</i>	<i>7.5</i>	<i>HK</i>	<i>2.0</i>	<i>22.5</i>	<i>11.3</i>
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<p>(v) Column axial forces: (<i>Consider the Fig. A .2</i>)</p> <p><i>Joint B:</i></p> $\Sigma F_V = 0$ $F_{AB} = F_{BC} + V_{BE} = 2.5 + 11.3 = 13.8 \text{ kN}$																			

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 6 of 6	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 1		
		Made by <i>SSSR</i>	Date 24-1-2000
	Checked by <i>PU</i>	Date 30-4-2000	

$$\text{Joint E: } F_{ED} = F_{EF} + V_{EH} - V_{EB} = -0.83 + 7.5 - 11.3 = -4.6 \text{ kN}$$

$$\text{Joint H: } F_{GH} = V_{EH} + F_{IH} - V_{HK} = 7.5 - 0.83 - 11.3 = -4.6 \text{ kN}$$

$$\text{Joint K: } F_{JK} = V_{KH} + F_{LK} = 11.3 + 2.5 = 13.8 \text{ kN}$$

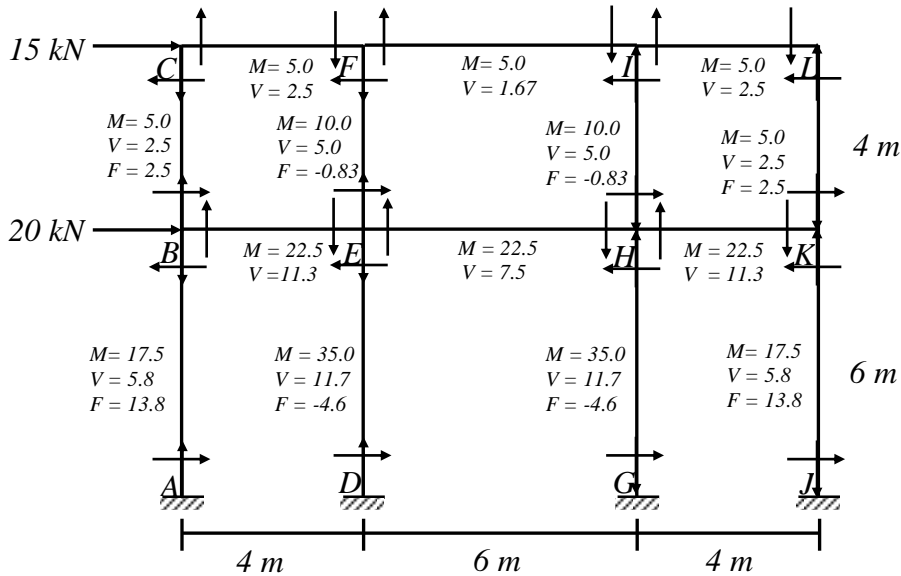


Fig. A. 2. Axial forces in columns and shear forces in members

M – Moment in kN-m
V – Shear in kN
F – Axial force in kN

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet <i>1 of 7</i>	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 2		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	

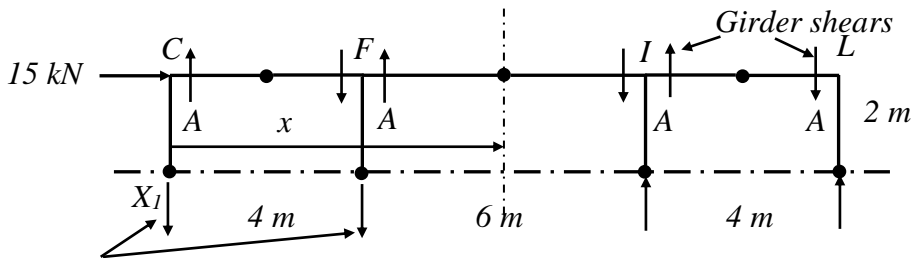
Problem 2:

Analyse the building frame shown in previous example (Fig. A) using Cantilever method. Assume cross-sectional areas of all the columns as equal.

Top storey:

(1) Location of centroidal line of columns of the storey:

Let the area of each column be A and x be distance to the centre of gravity of columns shown in Fig. B.1



Axial forces in columns

Fig. B.1

Take moments about column BC

$$x = \frac{0 \times A + 4 \times A + 10 \times A + 14 \times A}{4A} = \frac{28A}{4A} = 7m$$

(2) Column axial forces: (See Fig. B.1)

In cantilever method, it is assumed that the axial forces in the columns are proportional to the horizontal distance from the center of gravity of the columns in the storey.

Say, $F_{BC} = F$

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 2 of 7	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example – 2		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	
$\frac{F_{BC}}{F_{EF}} = \frac{7}{3} \Rightarrow F_{EF} = \frac{3}{7} F_{BC} = 3/7 F$ $F_{HI} = \frac{3}{7} F \quad ; \quad F_{KL} = F$ <p>Take moments about X_1,</p> $15 \times 2 + F_{EF} \times 4 - 10 \times F_{HI} - 14 F_{KL} = 0$ $15 \times 2 + \frac{3}{7} F \times 4 - 10 \times \frac{3}{7} F - 14 F = 0$ $30 - \frac{18}{7} F - 14 F = 0$ $30 = \frac{116}{7} F$ $F = \frac{210}{116} = 1.81 \text{ kN}$ $F_{BC} = 1.81 \text{ kN} \qquad F_{EF} = 0.78 \text{ kN}$ $F_{HI} = 0.78 \text{ kN} \qquad F_{KL} = 1.81 \text{ kN}$ <p>(3) Shear forces at the ends of beams: (See Fig. B.1)</p> <p>Joint C: $\sum F_y = 0 \Rightarrow V_{CF} = F_{BC} = 1.81 \text{ kN}$</p> <p>Joint F: $V_{FC} + F_{EF} = V_{FI}$ $V_{FI} = 1.81 + 0.78 = 2.59 \text{ kN} \quad (\because V_{FC} = V_{CF})$</p>			

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 3 of 7	Rev																																				
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<p>Joint I: $V_{IL} = V_{IF} - F_{HI} = 2.59 - 0.78 = 1.81 \text{ kN}$ ($V_{IF} = V_{FI}$)</p> <p>Joint L: $V_{LI} = F_{KL} = 1.81 \text{ kN}$</p> <p>(4) Girder moments:</p> <p>Girder moment = Girder shear * Span/2</p> <table border="1"> <thead> <tr> <th>Girder</th> <th>Shear (kN)</th> <th>Span /2 (m)</th> <th>Moment (kN-m)</th> </tr> </thead> <tbody> <tr> <td>CF</td> <td>1.81</td> <td>2.0</td> <td>3.62</td> </tr> <tr> <td>FI</td> <td>2.59</td> <td>3.0</td> <td>7.77</td> </tr> <tr> <td>IL</td> <td>1.81</td> <td>2.0</td> <td>3.62</td> </tr> </tbody> </table> <p>(5) Column moments: At each joint, sum of girder moments equals to sum of column moments. Consider joints from left corner of the floor.</p> <p>Joint C: $M_{CB} = M_{CF} = 3.62 \text{ kN-m}$</p> <p>Joint F: $M_{FE} = M_{FC} + M_{FI}$ $= 3.62 + 7.77 = 16.3 \text{ kN-m}$</p> <p>Joint I: $M_{IH} = M_{IF} + M_{IL}$ $= 7.77 + 3.62 = 16.3 \text{ kN-m}$</p> <p>Joint L: $M_{LI} = M_{LK} = 3.62 \text{ kN-m}$</p> <p>(6) Column Shears:</p> <p>Column Shear = Column moment / (Length/2)</p> <table border="1"> <thead> <tr> <th>Column</th> <th>Moment (kN-m)</th> <th>Length/2 (m)</th> <th>Shear (kN)</th> </tr> </thead> <tbody> <tr> <td>BC</td> <td>3.62</td> <td>2.0</td> <td>1.81</td> </tr> <tr> <td>EF</td> <td>11.4</td> <td>2.0</td> <td>5.70</td> </tr> <tr> <td>HI</td> <td>11.4</td> <td>2.0</td> <td>5.70</td> </tr> <tr> <td>KL</td> <td>3.62</td> <td>2.0</td> <td>1.81</td> </tr> </tbody> </table>				Girder	Shear (kN)	Span /2 (m)	Moment (kN-m)	CF	1.81	2.0	3.62	FI	2.59	3.0	7.77	IL	1.81	2.0	3.62	Column	Moment (kN-m)	Length/2 (m)	Shear (kN)	BC	3.62	2.0	1.81	EF	11.4	2.0	5.70	HI	11.4	2.0	5.70	KL	3.62	2.0	1.81
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<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 4 of 7	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 2		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	

Ground storey:

(i) Location of centroidal line of columns of the storey:

Consider the following free body diagram shown Fig. B.2

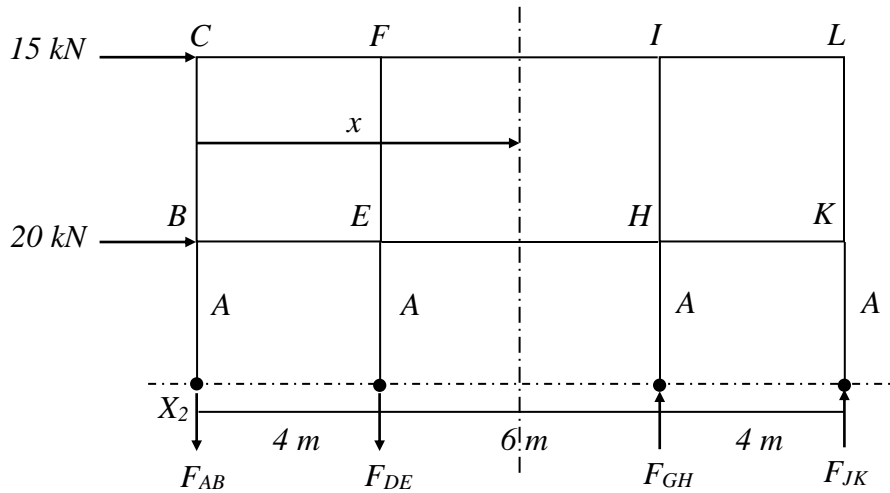


Fig. B. 2

Let the area of each column be 'A' and x be the distance to the centre of gravity of columns as shown in Fig. B.2

$$x = \frac{0 \times A + 4 \times A + 10 \times A + 14 \times A}{4A} = 7 \text{ m}$$

(ii) Column axial forces:

Say, $F_{AB} = F$

Then, $F_{DE} = 3/7 F$; $F_{GA} = 3/7 F$ and $F_{JK} = F$

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 5 of 7	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 2		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	
<p>Taking moments about X₂,</p> $15 \times (4 + 6/2) + 20 \times (6/2) + F_{DE} \times 4 - F_{GH} \times 10 - 14 \times F_{JK} = 0$ $\Rightarrow 105 + 60 + \frac{12}{7}F - \frac{30}{7}F - 14F = 0$ $\Rightarrow F = \frac{165 \times 7}{116} = 10.0 \text{ kN}$ <p>Axial forces in columns are,</p> $F_{AB} = 10.0 \text{ kN} \quad ; \quad F_{DE} = 4.3 \text{ kN}$ $F_{GH} = 4.3 \text{ kN} \quad ; \quad F_{JK} = 10.0 \text{ kN.}$ <p>(iii) Beam shears: [See Fig B.3]</p> <p>Joint B: $F_{AB} = V_{BE} + F_{BC}$ $V_{BE} = 10.0 - 1.81 = 8.2$</p> <p>Joint E: $V_{EB} + F_{DE} = V_{EH} + F_{EF}$ $V_{EH} = 8.2 + 4.3 - 0.78$ $= 11.7 \text{ kN} \quad (V_{EB} = V_{BE})$</p> <p>Joint H: $V_{HK} = V_{HE} - F_{GH} + F_{HI}$ $V_{HK} = 11.7 - 4.3 + 0.78$ $= 8.2 \text{ kN} \quad (V_{HE} = V_{EH})$</p> <p>Joint K: $V_{KH} = F_{JK} - F_{KL} = 10.0 - 1.81$ $= 8.2 \text{ kN}$</p>			

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet <i>6 of 7</i>	Rev																																				
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<p>(iv) Girder Moments:</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">Span</th> <th style="text-align: left;">Length/2 (m)</th> <th style="text-align: left;">Shear (kN)</th> <th style="text-align: left;">Moment (kN-m)</th> </tr> </thead> <tbody> <tr> <td><i>BE</i></td> <td><i>2.0</i></td> <td><i>8.2</i></td> <td><i>16.4</i></td> </tr> <tr> <td><i>EH</i></td> <td><i>3.0</i></td> <td><i>11.7</i></td> <td><i>35.1</i></td> </tr> <tr> <td><i>HK</i></td> <td><i>2.0</i></td> <td><i>8.2</i></td> <td><i>16.4</i></td> </tr> </tbody> </table> <p>(v) Column moments:</p> <p><i>At each joint sum of column moments equals to sum of girder moments</i></p> <p>Joint B:</p> $M_{BC} + M_{BA} = M_{BE}$ $M_{BA} = M_{BE} - M_{BC}$ $= 16.4 - 3.62 = 12.8 \text{ kN-m}$ <p>Joint E:</p> $M_{EB} + M_{EH} = M_{EF} + M_{ED}$ $M_{ED} = M_{EB} + M_{EH} - M_{EF}$ $= 16.4 + 35.1 - 11.4 = 40.1 \text{ kN-m}$ <p>Joint H:</p> $M_{HE} + M_{HK} = M_{HI} + M_{HG}$ $M_{HG} = M_{HE} + M_{HK} - M_{HI}$ $= 35.1 + 16.4 - 11.4 = 40.1 \text{ kN-m}$ <p>Joint K:</p> $M_{KH} = M_{KL} + M_{JK}$ $M_{JK} = M_{KH} - M_{KL} = 16.4 - 3.62$ $= 12.8 \text{ kN-m}$ <p>(vi) Column shears:</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">Span</th> <th style="text-align: left;">Length/2 (m)</th> <th style="text-align: left;">Moment (kN-m)</th> <th style="text-align: left;">Shear (kN)</th> </tr> </thead> <tbody> <tr> <td><i>AB</i></td> <td><i>3.0</i></td> <td><i>12.8</i></td> <td><i>4.3</i></td> </tr> <tr> <td><i>DE</i></td> <td><i>3.0</i></td> <td><i>40.1</i></td> <td><i>13.3</i></td> </tr> <tr> <td><i>GH</i></td> <td><i>3.0</i></td> <td><i>40.1</i></td> <td><i>13.3</i></td> </tr> <tr> <td><i>JK</i></td> <td><i>3.0</i></td> <td><i>12.8</i></td> <td><i>4.3</i></td> </tr> </tbody> </table>				Span	Length/2 (m)	Shear (kN)	Moment (kN-m)	<i>BE</i>	<i>2.0</i>	<i>8.2</i>	<i>16.4</i>	<i>EH</i>	<i>3.0</i>	<i>11.7</i>	<i>35.1</i>	<i>HK</i>	<i>2.0</i>	<i>8.2</i>	<i>16.4</i>	Span	Length/2 (m)	Moment (kN-m)	Shear (kN)	<i>AB</i>	<i>3.0</i>	<i>12.8</i>	<i>4.3</i>	<i>DE</i>	<i>3.0</i>	<i>40.1</i>	<i>13.3</i>	<i>GH</i>	<i>3.0</i>	<i>40.1</i>	<i>13.3</i>	<i>JK</i>	<i>3.0</i>	<i>12.8</i>	<i>4.3</i>
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<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 7 of 7	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example – 2		
	Made by SSSR		Date 24-1-2000
		Checked by PU	Date 30-4-2000

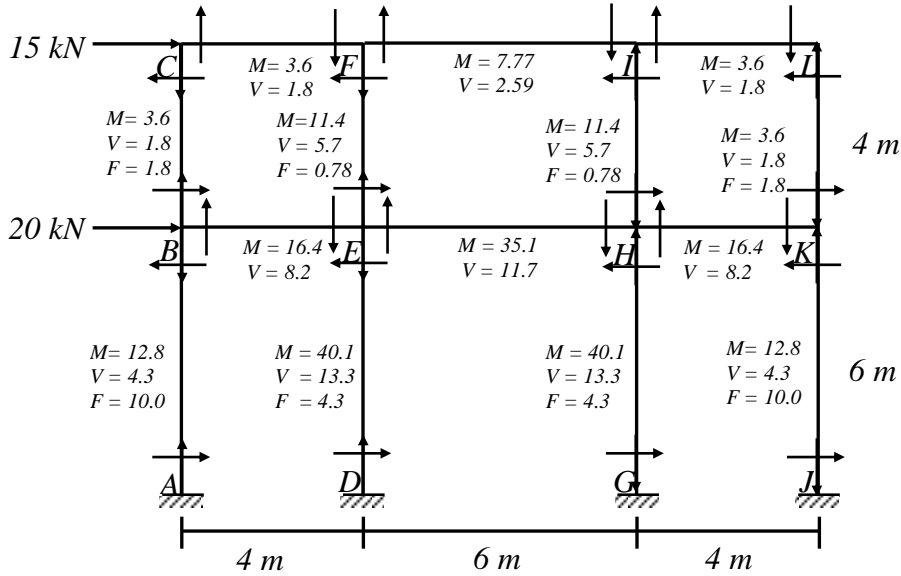


Fig. B. 3. Axial forces in columns and shear forces in members

M – Moment in kN-m
 V – Shear in kN
 F – Axial force in kN

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet <i>1 of 8</i>	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example – 3		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	

Problem 3:

Determine the moments at the ends of columns and beams in the building frame shown in Fig. C by factor method. The relative stiffness factors (k) are mentioned in figure.

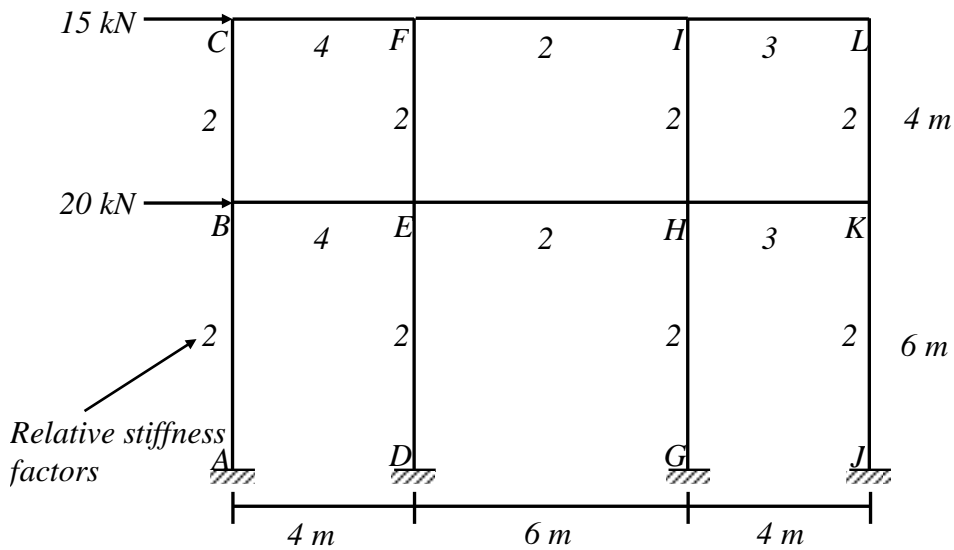


Fig. C

(1) Girder factors:

Girder factor, $g = \frac{\text{Sum of column relative stiffness factors at the joint}}{\text{Sum of total relative stiffness factors at that joint.}}$

Joint C:

$$g_C = \frac{2}{2+4} = 0.33$$

Joint F:

$$g_F = \frac{2}{4+2+2} = 0.25$$

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 2 of 8	Rev																														
	Job Title: MULTI-STOREYED BUILDINGS																																
	<i>Worked Example - 3</i>																																
		Made by SSSR	Date 24-1-2000																														
	Checked by PU	Date 30-4-2000																															
<p><i>Joint I:</i></p> $g_I = \frac{2}{2+2+3} = 0.286$ <p><i>Joint L:</i></p> $g_L = \frac{2}{2+3} = 0.4$ <p><i>Joint B:</i></p> $g_B = \frac{2+2}{2+2+4} = 0.5$ <p><i>Joint E:</i></p> $g_E = \frac{2+2}{2+2+2+4} = 0.4$ <p><i>Joint H:</i></p> $g_H = \frac{2+2}{2+2+2+3} = 0.444$ <p><i>Joint K:</i></p> $g_K = \frac{2+2}{2+2+3} = 0.571$ <p>(2) Column factors:</p> <p style="text-align: center;"><i>Column factor, c = 1-g</i></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th><i>Joint</i></th> <th><i>g</i></th> <th><i>c = (1-g)</i></th> <th><i>g/2</i></th> <th><i>c/2</i></th> </tr> </thead> <tbody> <tr> <td><i>C</i></td> <td><i>0.33</i></td> <td><i>0.67</i></td> <td><i>0.165</i></td> <td><i>0.335</i></td> </tr> <tr> <td><i>F</i></td> <td><i>0.25</i></td> <td><i>0.75</i></td> <td><i>0.125</i></td> <td><i>0.375</i></td> </tr> <tr> <td><i>I</i></td> <td><i>0.286</i></td> <td><i>0.714</i></td> <td><i>0.143</i></td> <td><i>0.357</i></td> </tr> <tr> <td><i>L</i></td> <td><i>0.4</i></td> <td><i>0.6</i></td> <td><i>0.2</i></td> <td><i>0.3</i></td> </tr> <tr> <td><i>B</i></td> <td><i>0.5</i></td> <td><i>0.5</i></td> <td><i>0.25</i></td> <td><i>0.25</i></td> </tr> </tbody> </table>				<i>Joint</i>	<i>g</i>	<i>c = (1-g)</i>	<i>g/2</i>	<i>c/2</i>	<i>C</i>	<i>0.33</i>	<i>0.67</i>	<i>0.165</i>	<i>0.335</i>	<i>F</i>	<i>0.25</i>	<i>0.75</i>	<i>0.125</i>	<i>0.375</i>	<i>I</i>	<i>0.286</i>	<i>0.714</i>	<i>0.143</i>	<i>0.357</i>	<i>L</i>	<i>0.4</i>	<i>0.6</i>	<i>0.2</i>	<i>0.3</i>	<i>B</i>	<i>0.5</i>	<i>0.5</i>	<i>0.25</i>	<i>0.25</i>
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Structural Steel Design Project	Job No:	Sheet 3 of 8	Rev			
	Job Title: MULTI-STOREYED BUILDINGS					
	<i>Worked Example - 3</i>					
		Made by SSSR	Date 24-1-2000			
Calculation Sheet		Checked by PU	Date 30-4-2000			
<i>Joint</i>	<i>g</i>	<i>c = (1-g)</i>	<i>g/2</i>	<i>c/2</i>		
<i>E</i>	0.4	0.6	0.2	0.3		
<i>H</i>	0.444	0.556	0.222	0.278		
<i>K</i>	0.571	0.429	0.285	0.215		
<i>A</i>	0	1.0	0	0.5		
<i>D</i>	0	1.0	0	0.5		
<i>G</i>	0	1.0	0	0.5		
<i>J</i>	0	1.0	0	0.5		
(3) Column and girder moment factors (C & G):						
<i>Joint</i>	<i>Members</i>	<i>c or g</i>	<i>Half values of factors of opposite end</i>	<i>(3) + (4)</i>	<i>k</i>	<i>C or G</i>
<i>(1)</i>	<i>(2)</i>	<i>(3)</i>	<i>(4)</i>	<i>(5)</i>	<i>(6)</i>	<i>(5) * (6)</i>
<i>C</i>	<i>CF</i>	0.33	0.125	0.455	4	1.82
	<i>CB</i>	0.67	0.25	0.92	2	1.84
<i>F</i>	<i>FE</i>	0.75	0.3	1.05	2	2.1
	<i>FI</i>	0.25	0.143	0.393	2	0.786
	<i>FC</i>	0.25	0.165	0.415	4	1.66
<i>I</i>	<i>IF</i>	0.286	0.125	0.411	2	0.822
	<i>IH</i>	0.714	0.278	0.992	2	1.984
	<i>IL</i>	0.286	0.2	0.486	3	1.458
<i>L</i>	<i>LI</i>	0.4	0.143	0.543	3	1.629
	<i>LK</i>	0.6	0.215	0.815	2	1.63
<i>B</i>	<i>BE</i>	0.5	0.2	0.7	4	2.8
	<i>BC</i>	0.5	0.335	0.835	2	1.67
	<i>BA</i>	0.5	0.5	1.0	2	2.0
<i>E</i>	<i>EF</i>	0.6	0.375	0.975	2	1.95
	<i>EB</i>	0.4	0.25	0.65	4	2.6
	<i>EH</i>	0.4	0.222	0.622	2	1.244
	<i>ED</i>	0.6	0.5	1.1	2	2.2

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet <i>4 of 8</i>	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 3		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	

Joint	Members	c or g	Half values of factors of opposite end	(3) + (4)	k	C or G
$A_1 = \left(\frac{\text{Total horizontal Shear of ground storey} \times \text{Height of ground storey}}{\Sigma C} \right)$						
(1)	(2)	(3)	(4) ΣC	(5)	(6)	(5)* (6)
H	HI	0.556	0.357	0.913	2	1.826
	HE	0.444	0.2	0.644	2	1.288
	HG	0.556	0.5	1.056	2	2.112
	HK	0.444	0.285	0.729	3	2.187
K	KL	0.429	0.3	0.729	2	1.458
	KH	0.571	0.222	0.793	3	2.379
	KJ	0.429	0.5	0.929	2	1.858
A	AB	1.0	0.25	1.25	2	2.5
D	DE	1.0	0.3	1.30	2	2.6
G	GH	1.0	0.278	1.278	2	2.556
J	JK	1.0	0.215	1.215	2	2.43

(4) Storey Constants:

For ground storey,

Let A_1 be the storey constant for determination of moments at the ends of columns of the ground storey. Then

Total horizontal shear of ground storey = 15+20 = 35 kN

Height of ground storey = 6 m

$$\Sigma C = (C_{AB} + C_{BA}) + (C_{ED} + C_{DE}) + (C_{HG} + C_{GH}) + (C_{KJ} + C_{JK})$$

$$= (2.5 + 2.0) + (2.2 + 2.6) + (2.112 + 2.556) + (1.858 + 2.43)$$

$$= 18.256$$

$$A_1 = \frac{35 \times 6}{18.256} = 11.5$$

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 5 of 8	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 3		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	
<p>For top storey,</p> <p>Let A_2 be the storey constant for determination of moments at the ends of columns of the top storey, then</p> $A_2 = \left(\frac{\text{Total horizontal Shear of top storey} \times \text{Height of top storey}}{\sum C} \right)$ <p>where, $\sum C$ = Sum of the column end moment factors of the storey.</p> <p>Total horizontal shear of top storey = 15 kN.</p> <p>Height of top storey = 4m.</p> $\begin{aligned} \sum C &= (C_{CB} + C_{BC}) + (C_{FE} + C_{EF}) + (C_{IH} + C_{HI}) + (C_{LK} + C_{KL}) \\ &= (1.84 + 1.67) + (2.1 + 1.95) + (1.984 + 1.826) + (1.63 + 1.458) \\ &= 14.458 \\ A_2 &= \frac{15 \times 4}{14.458} = 4.15 \end{aligned}$ <p>(5) Moments at the ends of columns:</p> <p>Ground storey moments:</p> <p>Moment at end of the column = Column moment factor at that end * A_1</p> $\begin{aligned} M_{AB} &= 2.5 * 11.5 = 28.7 \text{ kN-m} & M_{BA} &= 2.0 * 11.5 = 23.0 \text{ kN-m} \\ M_{DE} &= 2.6 * 11.5 = 29.9 \text{ kN-m} & M_{ED} &= 2.2 * 11.5 = 25.3 \text{ kN-m} \\ M_{GH} &= 2.556 * 11.5 = 29.4 \text{ kN-m} & M_{HG} &= 2.112 * 11.5 = 24.3 \text{ kN-m} \\ M_{JK} &= 2.43 * 11.5 = 27.9 \text{ kN-m} & M_{KJ} &= 1.858 * 11.5 = 21.4 \text{ kN-m} \end{aligned}$			

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 6 of 8	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	<i>Worked Example - 3</i>		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	
<p><i>Top storey moments:</i></p> <p><i>Moment at end of the column = Column moment factor at that end * A₂</i></p> <p>$M_{BC} = 1.67 * 4.15 = 6.93 \text{ kN-m}$ $M_{CB} = 1.84 * 4.15 = 7.64 \text{ kN-m}$</p> <p>$M_{EF} = 1.95 * 4.15 = 8.09 \text{ kN-m}$ $M_{FE} = 2.1 * 4.15 = 8.72 \text{ kN-m}$</p> <p>$M_{HI} = 1.826 * 4.15 = 7.58 \text{ kN-m}$ $M_{IH} = 1.984 * 4.15 = 8.23 \text{ kN-m}$</p> <p>$M_{KL} = 1.458 * 4.15 = 6.05 \text{ kN-m}$ $M_{LK} = 1.63 * 4.15 = 6.76 \text{ kN-m}$</p> <p>(6) Joint Constants:</p> <p><i>Joint constant (B) = $\frac{\text{Sum of column moments at the joint}}{\text{Sum of girder moment factors at that joint}}$</i></p> <p><i>For ground storey,</i></p> $B_B = \frac{M_{BC} + M_{BA}}{G_{BE}} = \frac{6.93 + 23.0}{2.8} = 10.69$ $B_E = \frac{M_{EF} + M_{ED}}{G_{EB} + G_{EH}} = \frac{8.09 + 25.3}{2.6 + 1.244} = 8.69$ $B_H = \frac{M_{HI} + M_{HG}}{G_{HE} + G_{HK}} = \frac{7.58 + 24.3}{1.288 + 2.187} = 9.17$ $B_K = \frac{M_{KL} + M_{KJ}}{G_{KH}} = \frac{6.05 + 21.37}{2.379} = 11.53$			

<h1>Structural Steel Design Project</h1> <h2>Calculation Sheet</h2>	Job No:	Sheet 7 of 8	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 3		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	
<p>For top storey,</p> $B_C = \frac{M_{CB}}{G_{CF}} = \frac{7.64}{1.82} = 4.20$ $B_F = \frac{M_{FE}}{G_{FC} + G_{FI}} = \frac{8.72}{1.66 + 0.786} = 3.56$ $B_I = \frac{M_{IH}}{G_{IF} + G_{IL}} = \frac{8.23}{0.822 + 1.458} = 3.61$ $B_L = \frac{M_{LK}}{G_{LI}} = \frac{6.76}{1.629} = 4.15$ <p>(7) Moments at the ends of beams:</p> <p>Moment at the end of beam equals to Girder moment factor at that end multiplied by respective joint constant.</p> $M_{CF} = 1.82 * 4.2 = 7.64 \text{ kN-m} ; M_{FC} = 1.66 * 3.56 = 5.91 \text{ kN-m}$ $M_{FI} = 0.786 * 3.56 = 2.8 \text{ kN-m} ; M_{IF} = 0.822 * 3.61 = 2.97 \text{ kN-m}$ $M_{IL} = 1.458 * 3.61 = 5.26 \text{ kN-m} ; M_{LI} = 1.629 * 4.15 = 6.76 \text{ kN-m}$ $M_{BE} = 2.8 * 10.69 = 29.9 \text{ kN-m} ; M_{EB} = 2.6 * 8.69 = 22.6 \text{ kN-m}$ $M_{EH} = 1.244 * 8.69 = 10.8 \text{ kN-m} ; M_{HE} = 1.288 * 9.17 = 11.8 \text{ kN-m}$ $M_{HK} = 1.332 * 9.17 = 12.2 \text{ kN-m} ; M_{KH} = 2.187 * 11.53 = 25.2 \text{ kN-m}$ <p>The values of girder factors and column factors are shown in Fig. C.2</p>			

<h1>Structural Steel Design Project</h1> <p>Calculation Sheet</p>	Job No:	Sheet <i>8 of 8</i>	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	<i>Worked Example - 3</i>		
		Made by <i>SSSR</i>	Date <i>24-1-2000</i>
	Checked by <i>PU</i>	Date <i>30-4-2000</i>	

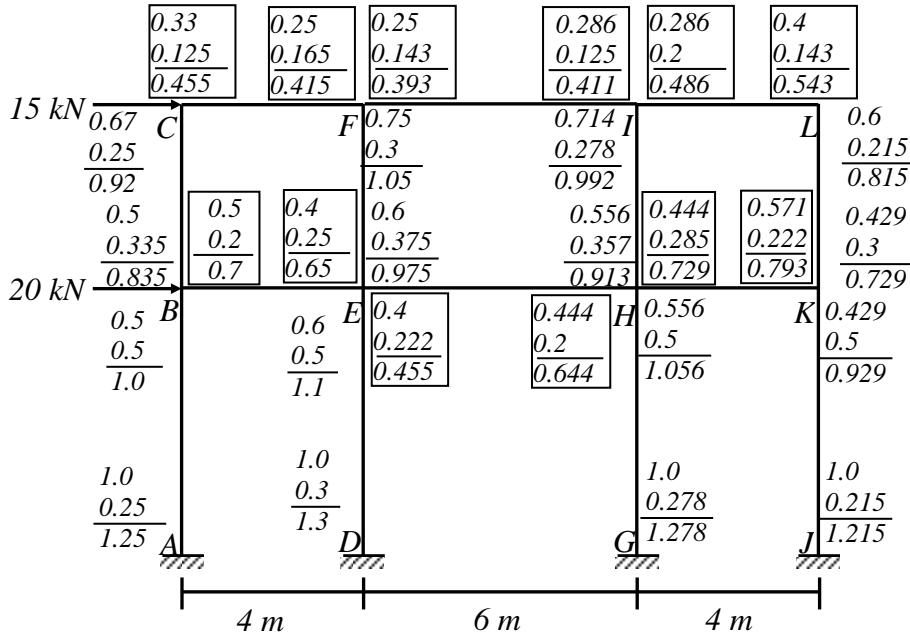


Fig. C. 2 Girder factors and column factors of the frame

Factors presented in rectangular boxes are girder factors and column factors are presented simply without any rectangular box.

<h1>Structural Steel Design Project</h1> <p>Calculation Sheet</p>	Job No:	Sheet <i>1 of 2</i>	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	<i>Worked Example - 4</i>		
		Made by <i>SSSR</i>	Date <i>24-1-2000</i>
	Checked by <i>PU</i>	Date <i>30-4-2000</i>	

Problem 4:

Determine the moments at the ends of columns and beams of the rigidly jointed building frame shown in Fig. D for the gravity load applied.

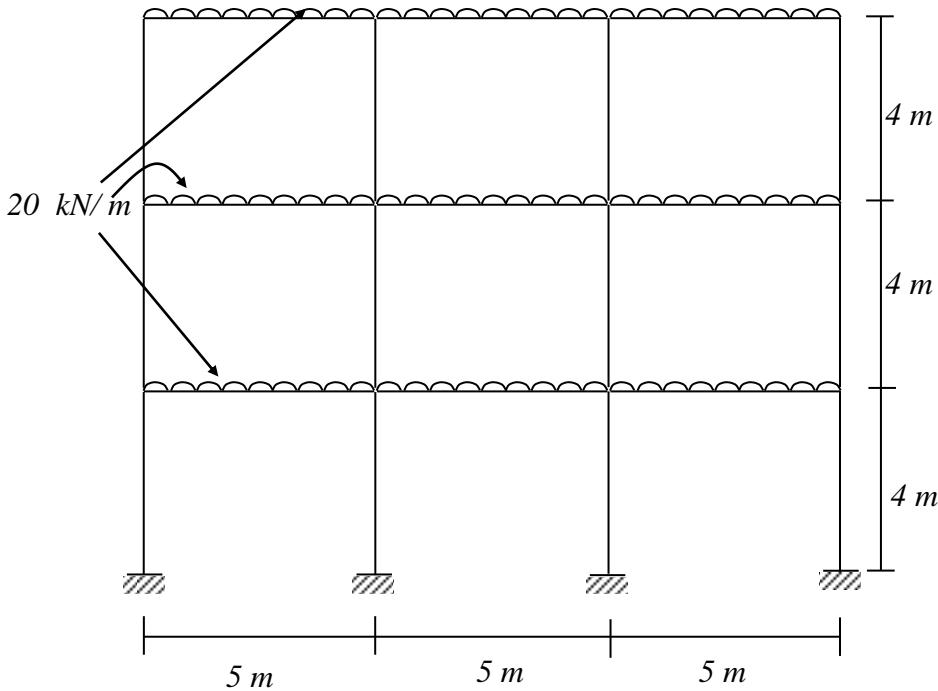
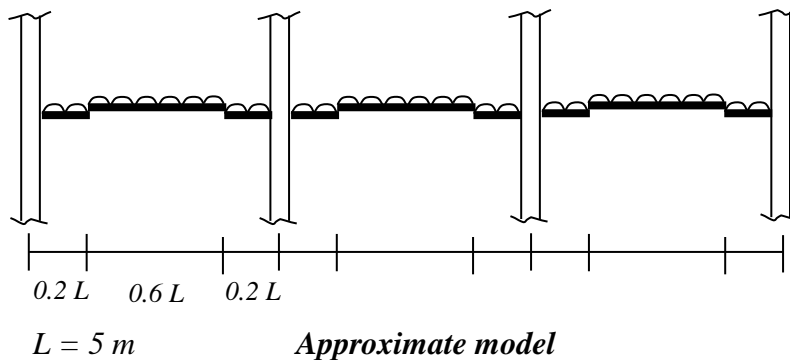


Fig. D

Consider the following approximate model.



<h1>Structural Steel Design Project</h1> <p>Calculation Sheet</p>	Job No:	Sheet 2 of 2	Rev
	Job Title: MULTI-STOREYED BUILDINGS		
	Worked Example - 4		
		Made by SSSR	Date 24-1-2000
	Checked by PU	Date 30-4-2000	
<p>Maximum + ve B.M. at mid-span = $\frac{wL^2}{8}$</p> <p style="margin-left: 150px;">$= 20 * 3^2 / 8 = 22.5 \text{ kN-m}$</p> <p>End reaction = $wl/2 = 20 * 3/2 = 30 \text{ kN}$</p> <p>Maximum negative B.M. at end column = $30 * 1 + (20 * 1 * 1) / 2$</p> <p style="margin-left: 150px;">$= 40 \text{ kN-m}$</p> <p>Bending moment in the interior column = $40 - 40 = 0$</p> <p>B.M. diagram for the frame:</p> 