26 STEEL-CONCRETE COMPOSITE COLUMNS-II

1.0 INTRODUCTION

In a previous chapter, the design of a steel-concrete composite column under axial loading was discussed. This chapter deals with the design of steel-concrete composite columns subjected to both axial load and bending. To design a composite column under combined compression and bending, it is first isolated from the framework, and the end moments which result from the analysis of the system as a whole are taken to act on the column under consideration. Internal moments and forces within the column length are determined from the structural consideration of end moments, axial and transverse loads. For each axis of symmetry, the buckling resistance to compression is first checked with the relevant non-dimensional slenderness of the composite column. Thereafter the moment resistance of the composite cross-section is checked in the presence of applied moment about each axis, e.g. x-x and y-y axis, with the relevant non-dimensional slenderness values of the composite column. For slender columns, both the effects of long term loading and the second order effects are included.

2.0 COMBINED COMPRESSION AND UNI-AXIAL BENDING

The design method described here is an extension of the simplified design method discussed in the previous chapter for the design of steel-concrete composite columns under axial load.

2.1 Interaction Curve for Compression and Uni-axial Bending

The resistance of the composite column to combined compression and bending is determined using an interaction curve. Fig. *1* represents the non-dimensional interaction curve for compression and uni-axial bending for a composite cross-section.

In a typical interaction curve of a column with steel section only, it is observed that the moment of resistance undergoes a continuous reduction with an increase in the axial load. However, a short composite column will often exhibit increases in the moment resistance beyond plastic moment under relatively low values of axial load. This is because under some favourable conditions, the compressive axial load would prevent concrete cracking and make the composite cross-section of a short column more effective in resisting moments. The interaction curve for a short composite column can be obtained by considering several positions of the neutral axis of the cross-section, h_n, and determining the internal forces and moments from the resulting stress blocks.

(It should be noted by way of contrast that *IS: 456-1978* for reinforced concrete columns specifies a *2 cm* eccentricity irrespective of column geometry. The method suggested here, using EC4, allows for an eccentricity of load application by the term α and therefore no further provision is necessary for steel columns. Another noteworthy feature is the prescription of strain limitation in *IS: 456-1978,* whereas EC4 does not impose such a limitation. The relevant provision in the Indian Code limits the concrete strain to *0.0035* minus *0.75* times the strain at the least compressed extreme fibre)

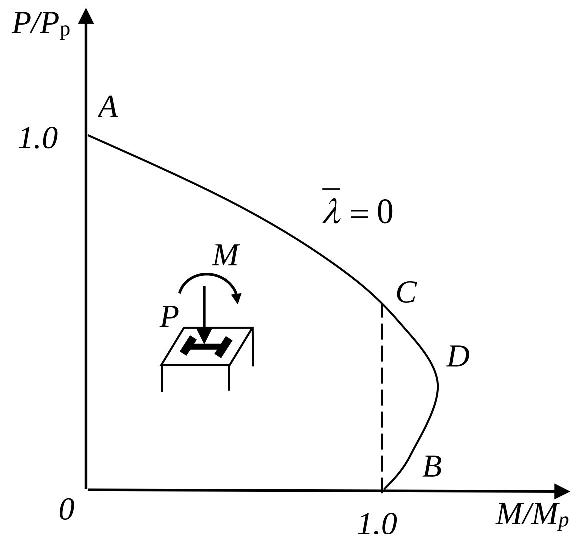

Fig. 1 Interaction curve for compression and uni-axial bending

Fig. *2* shows an interaction curve drawn using simplified design method suggested in the *UK* National Application Document for EC 4 *(NAD)*. This neglects the increase in moment capacity beyond M_P discussed above, (under relatively low axial compressive loads).

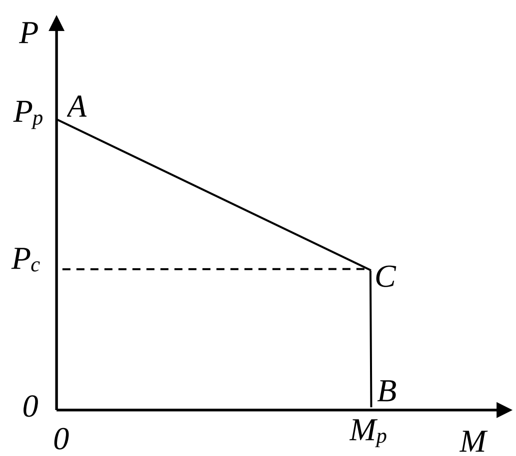

Fig. 2 Interaction curve for compression and uni-axial bending using the simplified method

Fig. *3* shows the stress distributions in the cross-section of a concrete filled rectangular tubular section at each point, *A, B* and *C* of the interaction curve given in Fig. *2*. It is important to note that:

- Point *A* marks the plastic resistance of the cross-section to compression (at this point the bending moment is zero).

$$P_A = P_p = A_a.f_y / \gamma_a + \alpha_c.A_{c.} (f_{ck})_{cy} / \gamma_c + A_s.f_{sk} / \gamma_s \quad (1)$$

$$M_A = 0 \quad (2)$$

- Point *B* corresponds to the plastic moment resistance of the cross-section (the axial compression is zero).

$$P_B = 0 \quad (3)$$

$$M_B = M_p = p_y (Z_{pa} - Z_{pan}) + p_{sk}(Z_{ps} - Z_{psn}) + p_{ck}(Z_{pc} - Z_{pcn}) \quad (4)$$

where

Z_{ps}, Z_{pa}, and Z_{pc} are plastic section moduli of the reinforcement, steel section, and concrete about their own centroids respectively.

Z_{psn}, Z_{pan} and Z_{pcn} are plastic section moduli of the reinforcement, steel section, and concrete about neutral axis respectively.

- At point *C*, the compressive and the moment resistances of the column are given as follows;

$$P_C = P_c = A_c\, p_{ck.} \quad (5)$$

$$M_C = M_p \quad (6)$$

- The expressions may be obtained by combining the stress distributions of the cross-section at points *B* and *C*; the compression area of the concrete at point *B* is equal to the tension area of the concrete at point *C*. The moment resistance at point *C* is equal to that at point *B*, since the stress resultants from the additionally compressed parts nullify each other in the central region of the cross-section. However, these additionally compressed regions create an internal axial force, which is equal to the plastic resistance to compression of the concrete, P_c alone.

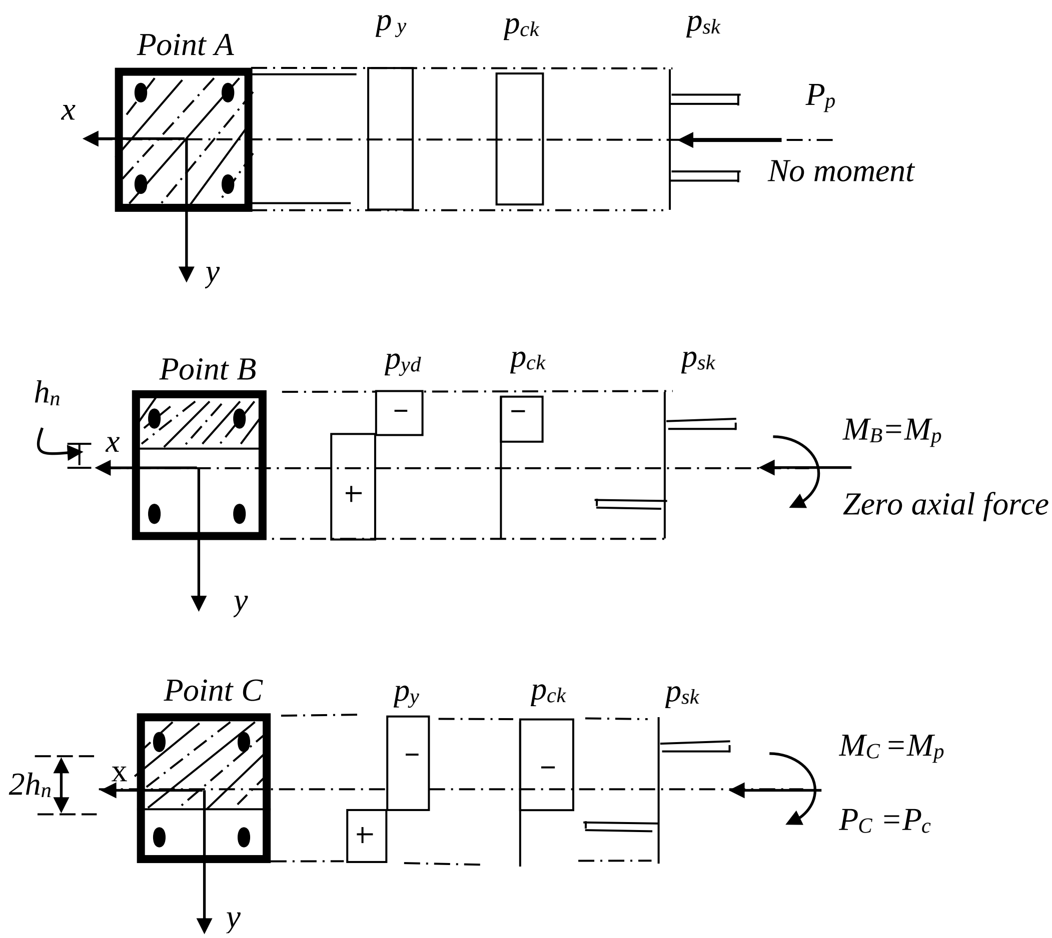

Fig. 3 Stress distributions for the points of the interaction curve for concrete filled rectangular tubular sections

It is important to note that the positions of the neutral axis for points *B* and *C*, h_n, can be determined from the difference in stresses at points *B* and *C*. The resulting axial forces, which are dependent on the position of the neutral axis of the cross-section, h_n, can easily be determined as shown in Fig. *4*. The sum of these forces is equal to P_c. This calculation enables the equation defining h_n to be determined, which is different for various types of sections.

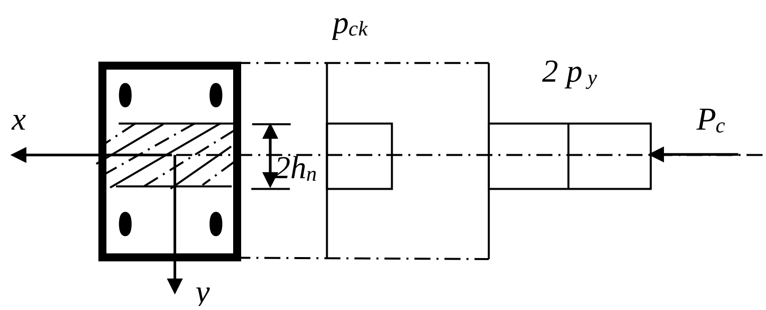

Fig. 4(a) Variation in the neutral axis positions

(1) **For concrete encased steel sections:**

<u>Major axis bending</u>

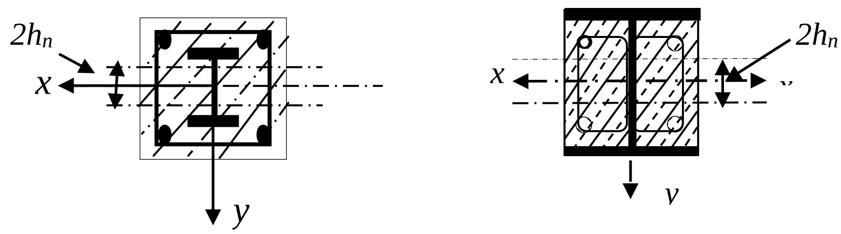

Fig. 4(b)

(1) Neutral axis in the web: $h_n \leq [\, h/2 - t_f \,]$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w (2p_y - p_{ck})}$$

(2) Neutral axis in the flange: $[h/2 - t_f\,] \leq h_n \leq h/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck}) + (b - t_w)(h - 2t_f)(2p_y - p_{ck})}{2b_c p_{ck} + 2b(2p_y - p_{ck})}$$

(3) Neutral axis outside the steel section: $h/2 \leq h_n \leq h_c/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck}) + A_a (2p_y - p_{ck})}{2b_c p_{ck}}$$

<u>Minor axis bending</u>

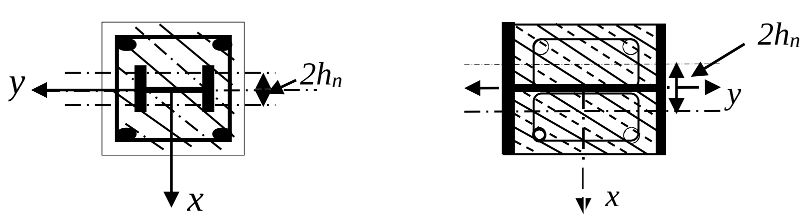

Fig. 4(c)

(1) Neutral axis in the web: $h_n \leq t_w/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2h_c p_{ck} + 2h(2p_y - p_{ck})}$$

(2) Neutral axis in the flange: $t_w/2 < h_n < b/2$

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck}) + t_w(2t_f - h)(2p_y - p_{ck})}{2h_c p_{ck} + 4t_f(2p_y - p_{ck})}$$

(3) Neutral axis outside the steel section: $b/2 \le h_n \le b_c/2$

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck}) - A_a(2p_y - p_{ck})}{2h_c p_{ck}}$$

Note: A'_s is the sum of the reinforcement area within the region of $2h_n$

(2) For concrete filled tubular sections

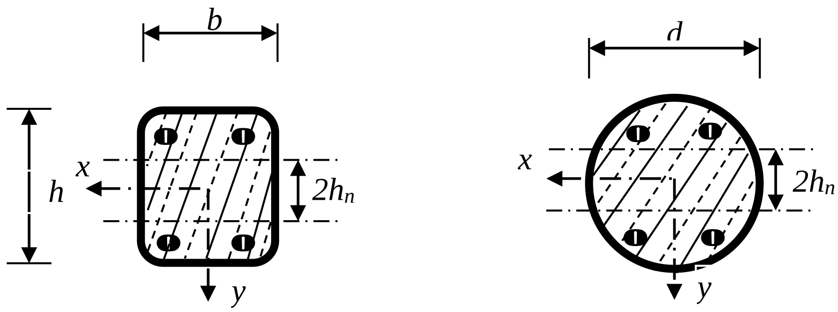

Fig. 4(d)

<u>Major axis bending</u>

$$h_n = \frac{A_c p_{ck} - A_s'(2p_{sk} - p_{ck})}{2b_c p_{ck} + 4t(2p_y - p_{ck})}$$

Note:

- For circular tubular section substitute $b_c = d$
- For minor axis bending the same equations can be used by interchanging h and b as well as the subscripts x and y.

2.2 Analysis of Bending Moments due to Second Order Effects

Under the action of a design axial load, P, on a column with an initial imperfection, e_o, as shown in Fig. *5*, there will be a maximum internal moment of $P.e_o$. It is important to note that this second order moment, or 'imperfection moment', does not need to be considered separately, as its effect on the buckling resistance of the composite column is already accounted for in the European buckling curves.

However, in addition to axial forces, a composite column may be also subject to end moments as a consequence of transverse loads acting on it, or because the composite column is a part of a frame. The moments and the displacements obtained initially are referred to as 'first order' values. For slender columns, the 'first order' displacements may be significant and additional or 'second order' bending moments may be induced under the actions of applied loads. As a simple rule, the second order effects should be considered if the buckling length to depth ratio of a composite column exceeds *15*.

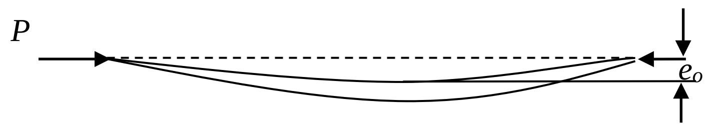

Fig. 5 Initially imperfect column under axial compression

The second order effects on bending moments for isolated non-sway columns should be considered if both of the following conditions are satisfied:

$$(1) \quad \frac{P}{P_{cr}} > 0.1 \tag{7}$$

where

P is the design applied load, and

P_{cr} is the elastic critical load of the composite column.

(2) Elastic slenderness conforms to:

$$\bar{\lambda} > 0.2 \tag{8}$$

where

$\bar{\lambda}$ is the non-dimensional slenderness of the composite column

In case the above two conditions are met, the second order effects may be allowed for by modifying the maximum first order bending moment (moment obtained initially), M_{max}, with a correction factor *k*, which is defined as follows:

$$k = \frac{1}{1-\frac{P}{P_{cr}}} \geq 1.0 \tag{9}$$

where

P is the applied design load.
P_{cr} is the elastic critical load of the composite column.

2.3 **Resistance of Members under Combined Compression and Uni-axial Bending**

The graphical representation of the principle for checking the composite cross-section under combined compression and uni-axial bending is illustrated in Fig. *6*.

The design checks are carried out in the following stages:

(1) The resistance of the composite column under axial load is determined in the absence of bending, which is given by $\chi\ P_p$. The procedure is explained in detail in the previous chapter.

(2) The moment resistance of the composite column is then checked with the relevant non-dimensional slenderness, in the plane of the applied moment. As mentioned before, the initial imperfections of columns have been incorporated and no additional consideration of geometrical imperfections is necessary.

The design is adequate when the following condition is satisfied:

$$M \le 0 \cdot 9 \mu\, M_p \tag{10}$$

where

M	is the design bending moment, which may be factored to allow for second order effects, if necessary
μ	is the moment resistance ratio obtained from the interaction curve.
M_p	is the plastic moment resistance of the composite cross-section.

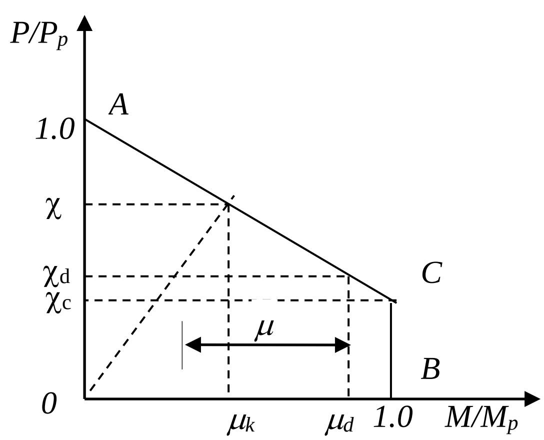

Fig. 6 Interaction curve for compression and uni-axial bending using the simplified method

The interaction curve shown in Fig. *6* has been determined without considering the strain limitations in the concrete. Hence the moments, including second order effects if necessary, are calculated using the effective elastic flexural stiffness, $(EI)_e$, and taking into account the entire concrete area of the cross-section, (i.e. concrete is uncracked).

Consequently, a reduction factor of *0.9* is applied to the moment resistance as shown in Equation *(10)* to allow for the simplifications in this approach. If the bending moment and the applied load are independent of each other, the value of μ must be limited to *1.0.*

Moment resistance ratio μ can be obtained from the interaction curve or may be evaluated. The method is described below.

Consider the interaction curve for combined compression and bending shown in Fig. *6*. Under an applied force P equal to χP_p, the horizontal coordinate $\mu_k\ M_p$ represents the second order moment due to imperfections of the column, or the 'imperfection moment'. It is important to recognise that the moment resistance of the column has been fully utilised in the presence of the 'imperfection moment'; the column cannot resist any additional applied moment.

χ_d represents the axial load ratio defined as follows:

$$\chi_d = P/P_p \tag{11}$$

By reading off the horizontal distance from the interaction curve, the moment resistance ratio, μ, may be obtained and the moment resistance of the composite column under combined compression and bending may then be evaluated.

In accordance with the *UK NAD*, the moment resistance ratio μ for a composite column under combined compression and uni-axial bending is evaluated as follows:

$$\mu = \frac{(\chi - \chi_d)}{(1-\chi_c)\chi} \qquad \text{when } \chi_d \geq \chi_c \tag{12}$$

$$= 1 - \frac{(1-\chi)\chi_d}{(1-\chi_c)\chi} \qquad \text{when } \chi_d < \chi_c \tag{13}$$

where

χ_c = axial resistance ratio due to the concrete, $\dfrac{P_c}{P_p}$

χ_d = design axial resistance ratio, $\dfrac{P}{P_p}$

χ = reduction factor due to column buckling

The expression is obtained from geometry consideration of the simplified interaction curve illustrated in Fig. *6*. A worked example illustrating the use of the above design procedure is appended to this chapter.

3.0 COMBINED COMPRESSION AND BI-AXIAL BENDING

For the design of a composite column under combined compression and bi-axial bending, the axial resistance of the column in the presence of bending moment for each axis has to be evaluated separately. Thereafter the moment resistance of the composite column is checked in the presence of applied moment about each axis, with the relevant non-dimensional slenderness of the composite column. Imperfections have to be considered only for that axis along which the failure is more likely. If it is not evident which plane is more critical, checks should be made for both the axes.

The moment resistance ratios μ_x and μ_y for both the axes are evaluated as given below:

$$\mu_x = \frac{(\chi_x - \chi_d)}{(1-\chi_c)\chi_x} \qquad \text{when } \chi_d \geq \chi_c \qquad (14)$$

$$= 1 - \frac{(1-\chi_x)\chi_d}{(1-\chi_c)\chi_x} \qquad \text{when } \chi_d < \chi_c \qquad (15)$$

$$\mu_y = \frac{(\chi_y - \chi_d)}{(1-\chi_c)\chi_y} \qquad \text{when } \chi_d \geq \chi_c \qquad (16)$$

$$= 1 - \frac{(1-\chi_y)\chi_d}{(1-\chi_c)\chi_y} \qquad \text{when } \chi_d < \chi_c \qquad (17)$$

where

χ_x and χ_y are the reduction factors for buckling in the *x* and *y* directions respectively.

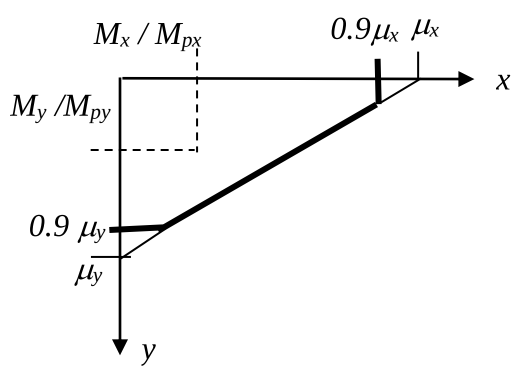

Fig. 7 Moment interaction curve for bi-axial bending

In addition to the two conditions given by Equations *(18)* and *(19)*, the interaction of the moments must also be checked using moment interaction curve as shown in Fig. *7*. The linear interaction curve is cut off at $0.9\mu_x$ and $0.9\mu_y$. The design moments, M_x and M_y related to the respective plastic moment resistances must lie within the moment interaction curve.

Hence the three conditions to be satisfied are:

$$\frac{M_x}{\mu_x M_{px}} \leq 0.9 \tag{18}$$

$$\frac{M_y}{\mu_y M_{py}} \leq 0.9 \tag{19}$$

$$\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \leq 1.0 \tag{20}$$

When the effect of geometric imperfections is not considered the moment resistance ratio is evaluated as given below:

$$\mu = \frac{(1-\chi_d)}{(1-\chi_c)} \qquad \text{when } \chi_d > \chi_c \tag{21}$$

$$= 1.0 \qquad \text{when } \chi_d \leq \chi_c \tag{22}$$

A worked example on combined compression and bi-axial bending is appended to this chapter.

4.0 STEPS IN DESIGN

4.1 Design Steps for columns with axial load and uni-axial bending

4.1.1 List the composite column specifications and the design values of forces and moments.

4.1.2 List material properties such as f_y, f_{sk}, $(f_{ck})_{cy}$, E_a, E_s, E_c

4.1.3 List section properties A_a, A_s, A_c, I_a, I_s, I_c of the selected section

4.1.4 Design checks

(1) Evaluate plastic resistance, P_p of the cross-section from equation,

$P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s$

(2) Evaluate effective flexural stiffness, $(EI)_e$ of the cross-section for short term loading in *x* and *y* direction using equation,

$(EI)_e = E_a I_a + 0.8\, E_{cd} I_c + E_s I_s$

(3) Evaluate non-dimensional slenderness, $\bar{\lambda}_x$ and $\bar{\lambda}_y$ in x and y directions from equation,

$$\bar{\lambda} = \left(\frac{P_{pu}}{(P_{cr})} \right)^{\frac{1}{2}}$$

where

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

Note: P_{pu} is the plastic resistance of the section with $\gamma_a = \gamma_c = \gamma_s = 1.0$

and $$P_{cr} = \frac{\pi^2 (EI)_e}{\ell^2}$$

(4) Check for long-term loading

The effect of long term loading can be neglected if following conditions are satisfied:

- Eccentricity, *e* given by

e = M/P $\geq$ *2 times the cross section dimension in the plane of bending considered*

- the non-dimensional slenderness $\bar{\lambda}$ in the plane of bending being considered exceeds the limits given in Table 6 of the previous chapter (Steel Concrete Composite Column-I)

(5) Check the resistance of the section under axial compression for both *x* and *y* axes.

Design against axial compression is satisfied if following condition is satisfied for both the axes:

$P < \chi P_p$

where

χ= reduction factor due to column buckling.

$$= \frac{1}{\left(\phi + \left\{\phi^2 - \bar{\lambda}^2\right\}^{\frac{1}{2}}\right)}$$

where

$$and\ \phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if following conditions are satisfied for the plane of bending under consideration:

- $P / P_{cr} \leq 0.1$
- $\bar{\lambda} \leq 0.2$

(7) Evaluate plastic moment resistance of the composite column about the plane of bending under consideration.

$$M_p = p_y\,(Z_{pa} - Z_{pan}) + 0.5\,p_{ck}\,(Z_{pc} - Z_{pcn}) + p_{sk}\,(Z_{ps} - Z_{psn})$$

where

Z_{ps}, Z_{pa}, and Z_{pc} are plastic section modulus of the reinforcement, steel section, and concrete about their own axes respectively.

Z_{psn},, Z_{pan}, and Z_{pcn} are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis respectively.

(8) Check the resistance of the composite column under combined axial compression and uni-axial bending

The design against combined compression and uni-axial bending is adequate if following condition is satisfied:

$$M \leq 0.9\,\mu\,M_P$$

where

M design bending moment

M_p plastic moment resistanc

μ moment resistance ratio

4.2 Design Steps for columns with axial load and bi-axial bending

4.2.1 List the composite column specifications and the design values of forces and moments.

4.2.2 List material properties such as f_y, f_{sk}, $(f_{ck})_{cy}$, E_a, E_s, E_c

4.2.3 List section properties A_a, A_s, A_c, I_a, I_s, I_c of the selected section.

4.2.4 Design checks

(1) Evaluate plastic resistance, P_p of the cross-section from equation,

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy}/\gamma_c + A_s f_{sk}/\gamma_s$$

(2) Evaluate effective flexural stiffness, $(EI)_{ex}$ and $(EI)_{ey}$, of the cross- section for short term loading from equation,

$$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$$

$$(EI)_{ey} = E_a I_{ay} + 0.8\, E_{cd} I_{cy} + E_s I_{sy}$$

(3) Evaluate non-dimensional slenderness, $\bar{\lambda}_x$ and $\bar{\lambda}_y$ from equation,

$$\bar{\lambda}_x = \left(\frac{P_{pu}}{(P_{cr})_x}\right)^{\frac{1}{2}}$$

$$\bar{\lambda}_y = \left(\frac{P_{pu}}{(P_{cr})_y}\right)^{\frac{1}{2}}$$

where

$$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$$

Note: P_{pu} is the plastic resistance of the section with $\gamma_a = \gamma_c = \gamma_s = 1.0$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

and $$(P_{cr})_y = \frac{\pi^2 (EI)_{ey}}{\ell^2}$$

(4) Check for long term loading.

The effect of long-term loading can be neglected if following conditions are satisfied:

- Eccentricity, e given by

$e = M / P \geq 2$ times cross section dimension in the plane of bending considered.
$e_x \geq 2b_c$
and $e_y \geq 2h_c$

- the non-dimensional slenderness $\bar{\lambda}$ in the plane of bending being considered exceeds the limits given in Table 6 of the previous chapter (Steel Concrete Composite Column –I).

(5) Check the resistance of the section under axial compression about both the axes.
Design against axial compression is satisfied if following conditions are satisfied:

$P < \chi_x P_p$

$P < \chi_y P_p$

where

$$\chi_x = \frac{1}{\left(\phi_x + \left\{\phi_x^{\,2} - \bar{\lambda}_x^{\,2}\right\}^{\frac{1}{2}}\right)}$$

$$and\ \phi_x = 0.5\left[1 + \alpha_x\left(\bar{\lambda}_x - 0.2\right) + \bar{\lambda}_x^{\,2}\right]$$

$$\chi_y = \frac{1}{\left(\phi_y + \left\{\phi_y^{\,2} - \bar{\lambda}_y^{\,2}\right\}^{\frac{1}{2}}\right)}$$

$$and\ \phi_y = 0.5\left[1 + \alpha_y\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^{\,2}\right]$$

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$P / (P_{cr})_x \leq 0.1$ for bending about x-x axis

$P / (P_{cr})_y \leq 0.1$ for bending about y-y axis

(7) Evaluate plastic moment resistance of the composite column under axial compression and bi-axial bending about both the axes.

About x-x axis

$M_{px} = [p_y (Z_{pa}-Z_{pan}) + 0.5\ p_{ck} (Z_{pc}-Z_{pcn}) + p_{sk} (Z_{ps}- Z_{psn})]_x$

where

M_{px} plastic moment resistance about x-x axis

Z_{psx}, Z_{pax}, and Z_{pcx} are plastic section modulus of the reinforcement, steel section, and concrete about their own axes in *x* direction respectively.
Z_{psn}, Z_{pan},and Z_{pcn} are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis in x direction respectively.

About y-y axis

$M_{py} =[p_y (Z_{pay}-Z_{pan}) + 0.5\ p_{ck} (Z_{pcy}-Z_{pcn}) + p_{sk} (Z_{psy}- Z_{psn})]_y$

where

M_{py} plastic moment resistance about y-y axis

Z_{psy}, Z_{pay}, and Z_{pcy} are plastic section moduli of the reinforcement, steel section, and concrete about their own axes in *y* direction respectively.

Z_{psn}, Z_{pan},and Z_{pcn} are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis in *y* direction respectively.

(8) Evaluate resistance of the composite column under combined axial compression and bi-axial bending

The design against combined compression and bi-axial bending is adequate if following conditions are satisfied:

(1) $M_x \le 0.9\ \mu_x\ M_{Px}$

(2) $M_y \le 0.9\ \mu_y\ M_{Py}$

$$(3)\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \le 1.0$$

where

μ_x and μ_y are the moment resistance ratios in the *x* and *y* directions respectively.

5.0 CONCLUSION

In this chapter the design of steel-concrete composite column subjected to axial load and bending is discussed. The use of interaction curve in the design of composite column subjected to both uni-axial bending and bi-axial bending is also described. Worked out example in each case is also appended to this chapter.

NOTATION

A	cross-sectional area
b	breadth of element
d	diameter, depth of element.
e	eccentricity of loading
e_o	initial imperfections
E	modulus of elasticity
$(EI)_e$	effective elastic flexural stiffness of a composite cross-section.
$(f_{ck})_{cu}$	characteristic compressive (cube) strength of concrete
$(f_{ck})_{cy}$	characteristic compressive (cylinder) strength of concrete, given by 0.80 times 28 days cube strength of concrete.
f_{sk}	characteristic strength of reinforcement
f_y	yield strength of steel
f_{ctm}	mean tensile strength of concrete
p_{ck}, p_y , p_{sk}	design strength of concrete, steel section and reinforcement respectively
h	height of element
h_n	depth of neutral axis from the middle line of the cross-section
I	second moment of area (with subscripts)
k	moment correction factor for second order effects
ℓ	buckling (or effective) length
L	length or span
M	moment (with subscripts)
P	axial force
M_p	plastic moment resistance of a cross-section
P_p	plastic resistance to compression of the cross section.
P_{pu}	plastic resistance to compression of the cross section with $\gamma_a = \gamma_c = \gamma_s = 1.0$
P_{cr}	elastic critical load of a column
P_c	axial resistance of concrete, $A_c\ p_{ck}$
t	thickness of element
Z_p	plastic section modulus

Greek letters

γ_f	partial safety factor for loads
γ	partial safety factor for materials (with subscripts)
γ^*_c	Reduction factor(1.35) used for reducing E_{cm} value
λ	slenderness ($\bar{\lambda}$ = non-dimensional slenderness)
ε	coefficient $\sqrt{250/f_y}$
α	imperfection factor
α_c	strength coefficient for concrete
χ	reduction factor buckling
χ_c	axial resistance ratio due to concrete, P_c/P_p
μ	moment resistance ratio

The subscripts to the above symbols are as follows:

a	structural steel
b	buckling
c	concrete
f	flange
k	characteristic value
s	reinforcement
w	web of steel section

Note-The subscript *x*, *y* denote the *x*-*x* and *y*-*y* axes of the section respectively. *x*-*x* denotes the major axes whilst *y*-*y* denotes the minor principal axes.

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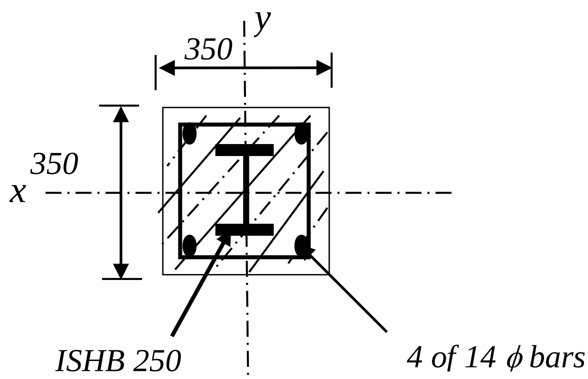
PROBLEM 1

Check the adequacy of the concrete encased composite section shown below for uni-axial bending.



4.1.1 DETAILS OF THE SECTION

Column dimension	*350 X 350 X 3000*
Concrete Grade	*M30*
Steel Section	*ISHB 250*
Steel Reinforcement	*4 Nos. of 14 mm dia bar, Fe415 grade*
Design Axial Load	*1500 kN*
Design bending moment about x-x axis	*180 kNm*
Design bending moment about y-y axis	*0 kNm*

Axial Load
P= 1500 kN

M_x *= 180 kN*

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DESIGN CALCULATIONS:

4.1.2 LIST MATERIAL PROPERTIES

(1)Structural steel

Steel section ISHB 250
Nominal yield strength $f_{y} =$ *250 N/mm*^{2}
Modulus of elasticity E_a *= 200 kN/mm*^{2}

(2) Concrete

Concrete grade M30
Characteristic strength $(f_{ck})_{cu}$ *=30 N/mm*^{2}
Secant modulus of elasticity for short term loading, E_{cm} *=31220 N/mm2*

(3) Reinforcing steel

Steel grade Fe 415
Characteristic strength f_{sk} *= 415N/mm*^{2}
Modulus of elasticity E_s *= 200 kN/mm*^{2}

(4) Partial safety factors

$\gamma_a = 1.15$
$\gamma_c = 1.5$
$\gamma_s = 1.15$

4.1.3 SECTION PROPERTIES OF THE GIVEN SECTION

(1) Steel section

$A_a = 6971\ mm^2$

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t_f = 9.7 mm; h= 250 mm ; t_w = 8.8 mm

$I_{ax} = 79.8 * 10^6\ mm^4$

$I_{ay} = 20.1 * 10^6\ mm^4$

$Z_{pax} = 699.8 * 10^3\ mm^3$

$Z_{pay} = 307.6 * 10^3\ mm^3$

(2) Reinforcing steel

4 bars of 14 mm dia, $A_s = 616\ mm^2$

(3) Concrete

$$Ac = A_{gross} - A_a - A_s$$
$$= 350 * 350 - 6971 - 616$$
$$= 114913\ mm^2$$

4.1.4 DESIGN CHECKS

(1) Plastic resistance of the section

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy}/\gamma_c + A_s f_{sk}/\gamma_s$$

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (0.80 *(f_{ck})_{cu}) / \gamma_c + A_s f_{sk}/\gamma_s$$

$$= [6971 * 250/1.15 + 0.85 * 114913 * 25/1.5 + 616 * 415/1.15]/1000$$
$$= 3366\ kN$$

$P_p = 3366\ kN$

(2) Effective elastic flexural stiffness of the section for short term loading

<u>*About the major axis*</u>

$$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$$

$$I_{ax} = 79.8 * 10^6\ mm^4$$

$E_{cd} = E_{cm}/\gamma^*_c = 31220/1.35 = 23125\ N/mm^2$

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$I_{sx} = Ah^2$
$= 616 * [\,350/2-25-7]^2$
$= 12.6 * 10^6\ mm^4$

$I_{cx} = (350)^4/12 - [\,79.8 + 12.6] * 10^6$
$= 1158 * 10^6\ mm^4$

$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$
$= 39.4 * 10^{12}\ N\ mm^2$

About minor axis

$(EI)_{ey} = 2.0 * 10^5 * 20.1 * 10^6 + 0.8 * 23125 * 1217.8 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$
$= 28.5 * 10^{12}\ N\ mm^2$

(3) Non dimensonal slenderness

$\bar{\lambda} = (P_{pu} / P_{cr})^{1/2}$

Value of P_{pu}:

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

$P_{pu} = A_a f_y + \alpha_c A_c * 0.80 * (f_{ck})_{cu} + A_s f_{sk}$

$= (6971 * 250 + 0.85 * 114913 * 25 + 415 * 616)/1000$

$= 4440\ kN$ — $P_{pu} = 4440\ kN$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

$$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2} = 43207\ kN$$

$(P_{cr})_x = 43207\ kN$

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$$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} \quad = 31254\,kN$$

$(P_{cr})_y = 31254$ kN

$\bar{\lambda}_x = (44.4 / 432.07)^{½} = 0.320$

$\bar{\lambda}_x = 0.320$

$\bar{\lambda}_y = (44.4 / 312.54)^{½} = 0.377$

$\bar{\lambda}_y = 0.377$

(4) Check for the effect of long term loading

The effect of long term loading can be neglected if anyone or both following conditions are satisfied:

- *Eccentricity, e given by*

  $e = M / P \geq 2$ *times the cross section dimension in the plane of bending considered.*

  $e_x = 180/1500$
  $= 0.12 < 2(0.35)$

  $e_y = 0$

- $\bar{\lambda} < 0.8$

Since condition (2) is satisfied, the influence of creep and shrinkage on the ultimate load need not be considered.

(5) Resistance of the composite column under axial compression

Design against axial compression is satisfied if following condition is satisfied:

$P < \chi P_p$

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Here,

$P = 1500\ kN$

$P_p = 3366\ kN$

and χ = *reduction factor for column buckling*

χ *values:*

<u>About major axis</u>

$\alpha_x = 0.34$

$\chi_x = 1 / \{\phi_x + (\phi_x^2 - \bar{\lambda}^2_x)^{½}\}$

$\phi_x = 0.5\,[1 + \alpha_x(\bar{\lambda}_x - 0.2) + \bar{\lambda}^2_x]$

$= 0.5\,[1 + 0.34(0.320 - 0.2) + (0.320)^2] = 0.572$

$\chi_x = 1 / \{0.572 + [(0.572)^2 - (0.326)^2]^{½}\}$

$= 0.956$

$\chi_x P_P > P$

$0.956 * 3366 = 3218\ kN > P\ (= 1500\ kN)$

<u>About minor axis</u>

$\alpha_y = 0.49$

$\phi_y = 0.5\,[1 + 0.49(0.377 - 0.2) + (0.377)^2]$
$= 0.61$

$\chi_y = 1 / \{0.61 + [(0.61)^2 - (0.377)^2]^{1/2}\}$
$= 0.918$

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$\chi_y P_P > P$

$0.918 * 3366 = 3090\ kN > P\ (=1500\ kN)$

$\therefore$ *The design is OK for axial compression.*

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$P / P_{cr} \leq 0.1$ *for major axis bending*

$1500/43207 = 0.035 < 0.1$

$\therefore$ *Check for second order effects is not necessary*

(7) Resistance of the composite column under axial compression and uni-axial bending

Compressive resistance of concrete, $P_c = A_c\, p_{ck}$
$=1628\ kN$

Plastic section modulus of the reinforcement

$Z_{ps} = 4(\pi / 4 * 14^2) * (350/2-25-14/2)$
$= 88 * 10^3\ mm^3$

Plastic section modulus of the steel section

$Z_{pa} = 699.8 * 10^3\ mm^3$

Plastic section modulus of the concrete

$Z_{pc} = b_c h_c^2 / 4 - Z_{ps} - Z_{pa}$
$= (350)^3/4 - 88 * 10^3 – 699.8 * 10^3$
$= 9931 * 10^3\ mm^3$

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Check that the position of neutral axis is in the web

$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w(2p_y - p_{ck})}$$

$$= \frac{114913 * \frac{0.85*25}{1.5}}{2*350*\frac{0.85*25}{1.5} + 2*8.8\left(2*\frac{250}{1.15} - \frac{0.85*25}{1.5}\right)}$$

$$= 93.99\ mm < \left(h/2 - t_f\right) = \left(\frac{250}{2} - 9.7\right) = 115.3\,mm$$

The neutral axis is in the web.

$A'_s = 0$ *as there is no reinforcement with in the region of the steel web*

<u>Section modulus about neutral axis</u>

$Z_{psn} = 0$ *(As there is no reinforcement with in the region of* $2h_n$ *from the middle line of the cross section)*

$$Z_{pan} = t_w h_n^2 = 8.8 * (93.99)^2$$
$$= 77740.3\ mm^3$$

$$Z_{pcn} = b_c h_n^2 - Z_{psn} - Z_{pan}$$
$$= 350\ (93.99)^2 - 77740. = 3014.2 * 10^3\ mm^3$$

Plastic moment resistance of section

$$M_p = p_y\ (Z_{pa} - Z_{pan}) + 0.5\ p_{ck}\ (Z_{pc} - Z_{pcn}) + p_{sk}\ (Z_{ps} - Z_{psn})$$

$$= 217.4\ (699800 - 77740) + 0.5 * 0.85 * 25/1.5\ (9931000 - 3014200) + 361\ (88 * 1000)$$

$$= 216\ kNm$$

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(8) Check of column resistance against combined compression and uni-axial bending

The design against combined compression and uni-axial bending is adequate if following condition is satisfied:

$M \leq 0.9\,\mu\,M_P$

$M = 180$ kNm

$M_p = 216$ kNm

μ = moment resistance ratio

$= 1 - \{(1-\chi)\,\chi_d\}/\{(1-\chi_c)\,\chi\}$

$= 1 - \{(1 - 0.956)\,0.446\}/\{(1 - 0.484)\,0.956\}$

$= 0.960$

$\therefore M < 0.9\,\mu\,M_p$

$< 0.9\,(0.960) * (216)$

< 187 kNm

Hence the composite column is acceptable and the check is satisfied.

$\chi_d = P / P_p$

$= 1500/3366$

$= 0.446$

$\chi_c = P_c/P_p$

$= 1628/3366$

$= 0.484$

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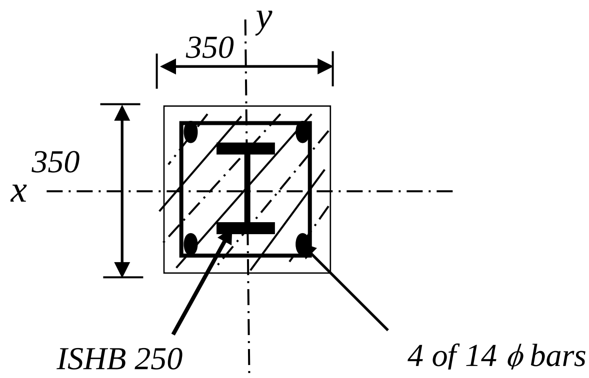
PROBLEM 2

Check the adequacy of the concrete encased composite section shown below for bi-axial bending



4.2.1 DETAILS OF THE SECTION

Column dimension	*350 X 350 X 3000*	
Concrete Grade	*M30*	
Steel Section	*ISHB 250*	
Steel Reinforcement	*Fe415*	
	4 Nos. of 14 mm dia bar	
Design Axial Load	*1500 kN*	*Axial Load = 1500kN*
Design bending moment about x-x axis	*180 kNm*	M_x *=180kNm*
Design bending moment about y-y axis	*120 kNm*	M_y *=120kNm*

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DESIGN CALCULATIONS:

4.2.2 LIST MATERIAL PROPERTIES

(1)Structural steel

Steel section ISHB 250
Nominal yield strength $f_y = 250\ N/mm^2$
Modulus of elasticity $E_a = 200\ kN/mm^2$

Concrete

Concrete grade M30
Characteristic strength $(f_{ck})_{cu} = 30\ N/mm^2$
Secant modulus of elasticity for short term loading, $E_{cm} = 31220\ N/mm^2$

Reinforcing steel

Steel grade Fe 415
Characteristic strength $f_{sk} = 415\ N/mm^2$
Modulus of elasticity $E_s = 200\ kN/mm^2$

Partial safety factors

$\gamma_a = 1.15$
$\gamma_c = 1.5$
$\gamma_s = 1.15$

4.2.3 LIST SECTION PROPERTIES OF THE GIVEN SECTION

(1) Steel section

$A_a = 6971\ mm^2$

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$t_f = 9.7$ mm
$h = 250$ mm
$t_w = 8.8$ mm
$I_{ax} = 79.8 * 10^6\ mm^4$
$I_{ay} = 20.1 * 10^6\ mm^4$
$Z_{px} = 699.8 * 10^3\ mm^3$
$Z_{py} = 307.6 * 10^3\ mm^3$

(2) Reinforcing steel

4 bars of 14 mm dia, $A_s = 616\ mm^2$

(3)Concrete

$$Ac = A_{gross} - A_a - A_s$$
$$= 350 * 350 - 6971 - 616$$
$$= 114913\ mm^2$$

4.2.4 DESIGN CHECKS

(1) Plastic resistance of the section

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c (f_{ck})_{cy}/\gamma_c + A_s f_{sk}/\gamma_s$$

$$P_p = A_a f_y/\gamma_a + \alpha_c A_c\, 0.80*(f_{ck})_{cu}/\gamma_c + A_s f_{sk}/\gamma_s$$
$$= [6971 * 250/1.15 + 0.85 * 114913 * 25/1.5 + 616 * 415/1.15]/1000$$
$$= 3366\ kN$$

$P_p = 3366\ kN$

(3) Effective elastic flexural stiffness of the section for short term loading

<u>About the major axis</u>

$$(EI)_{ex} = E_a I_{ax} + 0.8\, E_{cd} I_{cx} + E_s I_{sx}$$

$$I_{ax} = 79.8 * 10^6\ mm^4$$

$E_{cd} = E_{cm}/\gamma^*_c = 31220/1.35 = 23125 N/mm^2$

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$I_{sx} = Ah^2$

$= 616 * [350/2-25-7]^2$

$= 12.6 * 10^6\ mm^4$

$I_{cx} = (350)^4/12 - [79.8 + 12.6] * 10^6$

$= 1158 * 10^6\ mm^4$

$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$

$= 39.4 * 10^{12}\ N\ mm^2$

<u>About minor axis</u>

$(EI)_{ey} = 2.0 * 10^5 * 20.1 * 10^6 + 0.8 * 23125 * 1217.8 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$

$= 28.5 * 10^{12}\ N\ mm^2$

(4) Non dimensonal slenderness

$\bar{\lambda} = (P_{pu}/P_{cr})^{1/2}$

Value of P_{pu}:

$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$

$P_{pu} = A_a f_y + \alpha_c A_c\, 0.80 * (f_{ck})_{cu} + A_s f_{sk}$

$= (6971 * 250 + 0.85 * 114913 * 25 + 415 * 616)/1000$

$= 4440\ kN$

$P_{pu} = 4440\ kN$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

$$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2}$$

$= 43207\ kN$

$(P_{cr})_x = 43207\ kN$

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$$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} = 31254 kN$$

$\bar{\lambda}_x = (44.4 / 432.07)^{½} = 0.320$

$\bar{\lambda}_y = (44.4/312.54)^{½} = 0.377$

(5) Check for the effect of long term loading

The effect of long term loading can be neglected if anyone or both following conditions are satisfied:

- *Eccentricity, e given by*
  $e = M / P \geq 2$ *times the cross section dimension in the plane of bending considered)*
  $e_x = 180 / 1500$
  $= 0.12 < 2(0.350)$
  $e_y = 120/1500$
  $= 0.08 < 2 (0 .350)$
- $\bar{\lambda} < 0.8$

Since condition (2) is satisfied, the influence of creep and shrinkage on the ultimate load need not be considered.

(6) Resistance of the composite column under axial compression

Design against axial compression is satisfied if following condition is satisfied:

$P < \chi P_p$

$(P_{cr})_y$ = 31254 kN

$\bar{\lambda}_x = 0.320$

$\bar{\lambda}_y = 0.377$

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Here,

$P = 1500$ kN

$P_p = 3366$ kN

and χ = *reduction factor for column buckling*

χ *values:*

<u>About major axis</u>

$\alpha_x = 0.34$

$\chi_x = 1 / \{\phi + (\phi^2 - \bar{\lambda}^2{}_x)^{½}\}$

$\phi_x = 0.5\,[1 + \alpha_x(\bar{\lambda}_x - 0.2) + \bar{\lambda}^2{}_x]$

$= 0.5\,[1 + 0.34(0.320-0.2) + (0.320)^2] = 0.572$

$\chi_x = 1 / \{0.572 + [(0.572)^2 - (0.320)^2]^{½}\}$
$= 0.956$

$\chi_x P_{Px} > P$

$0.956 * 3366 = 3218$ kN $> P\ (=1500$ kN$)$

<u>About minor axis</u>

$\alpha_y = 0.49$

$\phi_y = 0.5\,[1 + 0.49(0.377 - 0.2) + (0.377)^2]$
$= 0.61$

$\chi_y = 1 / \{0.61 + [(0.61)^2 - (0.377)^2]^{1/2}\}$
$= 0.918$

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$\chi_y P_{py} > P$

$0.918 * 3366 = 3090\ kN > P\ (=1500\ kN)$

$\therefore$ *The design is OK for axial compression.*

(7) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$P/(P_{cr})_x \leq 0.1$ *for major axis bending*

$1500/43207 = 0.035 \leq 0.1$

$P/(P_{cr})_y \leq 0.1$ *for minor axis bending*

$1500/31254 = 0.048 \leq 0.1$

$\therefore$ *Check for second order effects is not necessary*

(8) Resistance of the composite column under axial compression and bi-axial bending

Compressive resistance of concrete, $P_c = A_c\, p_{ck}$

$= 1628\ kN$

About Major axis

Plastic section modulus of the reinforcement

$Z_{ps} = 4(\pi/4 * 14^2) * (350/2 - 25 - 14/2)$

$= 88 * 10^3\ mm^3$

Plastic section modulus of the steel section

$Z_{pa} = 699.8 * 10^3\ mm^3$

Plastic section modulus of the concrete

$Z_{pc} = b_c h_c^2/4 - Z_{ps} - Z_{pa}$

$= (350)^3/4 - 88 * 10^3 - 699.765 * 10^3$

$= 9931 * 10^3\ mm^3$

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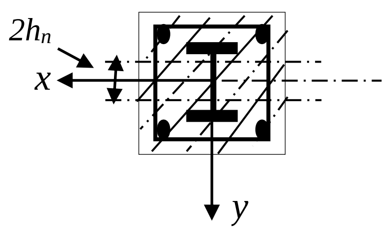
Check that the position of neutral axis is in the web



$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w(2p_y - p_{ck})}$$

$$= \frac{114913 * \dfrac{0.85 * 25}{1.5}}{2 * 350 * \dfrac{0.85 * 25}{1.5} + 2 * 8.8\left(2 * \dfrac{250}{1.15} - \dfrac{0.85 * 25}{1.5}\right)}$$

$$= 93.99\ mm \langle \left(h/2 - t_f\right) = \left(\frac{250}{2} - 9.7\right) = 115.3\,mm$$

The neutral axis is in the web

A'_s= 0 as there is no reinforcement with in the region of the steel web

<u>Section modulus about neutral axis</u>

Z_{psn} =0 (As there is no reinforcement with in the region of $2h_n$ from the middle line of the cross section)

Z_{pan} = $t_w\, h_n^2$ =8.8 * $(93.99)^2$
= 77740.3 mm^3

Z_{pcn} = $b_c h_n^2$ - Z_{psn} - Z_{pan}
= 350 $(93.99)^2$-77740.3
= 3014.2* 10^3 mm^3

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<tr><td colspan="3">Worked Example 2</td></tr>
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Plastic moment resistance of section

$M_p = p_y\,(Z_{pa}-Z_{pan}) + 0.5\,p_{ck}\,(Z_{pc}-Z_{pcn}) + p_{sk}\,(Z_{ps}-Z_{psn})$

$= 217.4\,(699800 - 77740.3) + 0.5 * 0.85 * 25/1.5\,(9931000 - 3014200) + 361\,(88 * 1000)$

$= 216$ kNm

<u>About minor axis</u>

Plastic section modulus of the reinforcement

$Z_{ps} = 4(\pi / 4 * 14^2) * (350/2 - 25 - 14/2)$

$= 88 * 10^3\ mm^3$

Plastic section modulus of the steel section

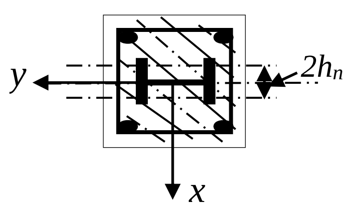
$Z_{pa} = 307.6 * 10^3\ mm^3$

Plastic section modulus of the concrete

$Z_{pc} = b_c h_c^2 / 4 - Z_{ps} - Z_{pa}$

$= (350)^3/4 - 88 * 10^3 - 307.6 * 10^3$

$= 10323 * 10^3\ mm^3$



$$h_n = \frac{A_c p_{ck} - A'_s(2p_{sk} - p_{ck}) + t_w(2t_f - h)(2p_y - p_{ck})}{2h_c p_{ck} + 4t_f(2p_y - p_{ck})}$$

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$$h_n = \frac{114913*14.2+8.8(2*9.7-250)(2*218-14.2)}{2*350*14.2+4*9.7\,(\,2*218-14.2\,)}$$

$$= 29.5\,mm \left({t_w}/{2} < h_n < {b}/{2} \right) = {8.8}/{2} < h_n < {250}/{2}$$

A'_s= 0 as there is no reinforcement with in the region of the steel web

Section modulus about neutral axis

Z_{psn} =0 (As there is no reinforcement with in the region of $2h_n$ from the middle line of the cross section)

$Z_{pan} = 2t_f\, h_n^2+(h-2t_f)/4*t_w^2$
$= 2(9.7)(29.5)^2 + [\{250-2(9.7)\}/4]*8.8^2$
$=21.3*10^3\ mm^3$

$Z_{pcn} = h_c h_n^2 - Z_{psn} - Z_{pan}$
$= 350\,(29.5)^2 - 21.3*10^3$
$=283.3 * 10^3\ mm^3$

$M_{py} = p_y\,(Z_{pa} - Z_{pan}) + 0.5\, p_{ck}\,(Z_{pc} - Z_{pcn}) + p_{sk}\,(Z_{ps} - Z_{psn})$

$= 217.4\,(307.589 - 21.3)*10^3 + 0.5 * 14.2 * (10323 - 283.3)*10^3 + 361\,(88 * 1000)$

$=165\ kNm$

(9) *Check of column resistance against combined compression and bi-axial bending*

The design against combined compression and bi-axial bending is adequate if following conditions are satisfied:

(1) $M \le 0.9\, \mu\, M_P$

About major axis

$M_x = 180\ kNm$

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M_{px} =216 kNm

μ_x = moment resistance ratio

$= 1- \{(1 - \chi_x)\ \chi_d\}/\{(1 - \chi_c)\ \chi_x\}$

$= 1- \{(1 – 0.956)\ 0.446\}/\{(1 – 0.484)\ 0.956\}$

$= 0.960$

$\therefore M_x < 0.9\ \mu_x\ M_{px}$

$< 0.9\ (0.960) * (216)$

$= 187$ kNm

<u>About minor axis</u>

M_y = 120 kNm

M_{py} =165 kNm

$\mu_y = 1- \{(1 - \chi_y)\ \chi_d\}/\{(1 - \chi_c)\ \chi_y\}$

$= 1- \{(1 – 0.918)\ 0.446\}/\{(1 – 0.448)\ 0.918\}$

$= 0.928$

$\therefore M_y < 0.9\ \mu_y\ M_{py}$

$< 0.9\ (0.928) * (165)$

<138 kNm (M_y=120 kN)

(2) $$\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \le 1.0$$

$$\frac{180}{0.960*216} + \frac{120}{0.928*165} > 1.0$$

Since design check (2) is not satisfied, the composite column is not acceptable.

$\chi_d = P/P_p$
$=1500/3366$
$=0.446$

$\chi_c = P_c/P_p$
$=1628\ /3366$
$=0.484$