21 COMPOSITE BEAMS – I

1.0 INTRODUCTION

In conventional composite construction, concrete slabs rest over steel beams and are supported by them. Under load these two components act independently and a relative slip occurs at the interface if there is no connection between them. With the help of a deliberate and appropriate connection provided between the beam and the concrete slab, the slip between them can be eliminated. In this case the steel beam and the slab act as a "***composite beam***" and their action is similar to that of a monolithic Tee beam. Though steel and concrete are the most commonly used materials for composite beams, other materials such as pre-stressed concrete and timber can also be used. Concrete is stronger in compression than in tension, and steel is susceptible to buckling in compression. By the composite action between the two, we can utilise their respective advantages to the fullest extent. Generally in steel-concrete composite beams, steel beams are integrally connected to prefabricated or cast in situ reinforced concrete slabs. There are many advantages associated with steel concrete composite construction. Some of these are listed below:

- The most effective utilisation of steel and concrete is achieved.
- Keeping the span and loading unaltered; a more economical steel section (in terms of depth and weight) is adequate in composite construction compared with conventional non-composite construction.
- As the depth of beam reduces, the construction depth reduces, resulting in enhanced headroom.
- Because of its larger stiffness, composite beams have less deflection than steel beams.
- Composite construction provides efficient arrangement to cover large column free space.
- Composite construction is amenable to *"fast-track"* construction because of using rolled steel and pre-fabricated components, rather than cast-in-situ concrete.
- Encased steel beam sections have improved fire resistance and corrosion.

2.0 ELASTIC BEHAVIOUR OF COMPOSITE BEAMS

The behaviour of composite beams under transverse loading is best illustrated by using two identical beams, each having a cross section of $b \times h$ and spanning a distance of ℓ, one placed at the top of the other. The beams support a uniformly distributed load of *w/*unit length as shown in Fig 1. For theoretical explanation, two extreme cases of no interaction and 100% (full) interaction are analysed below:

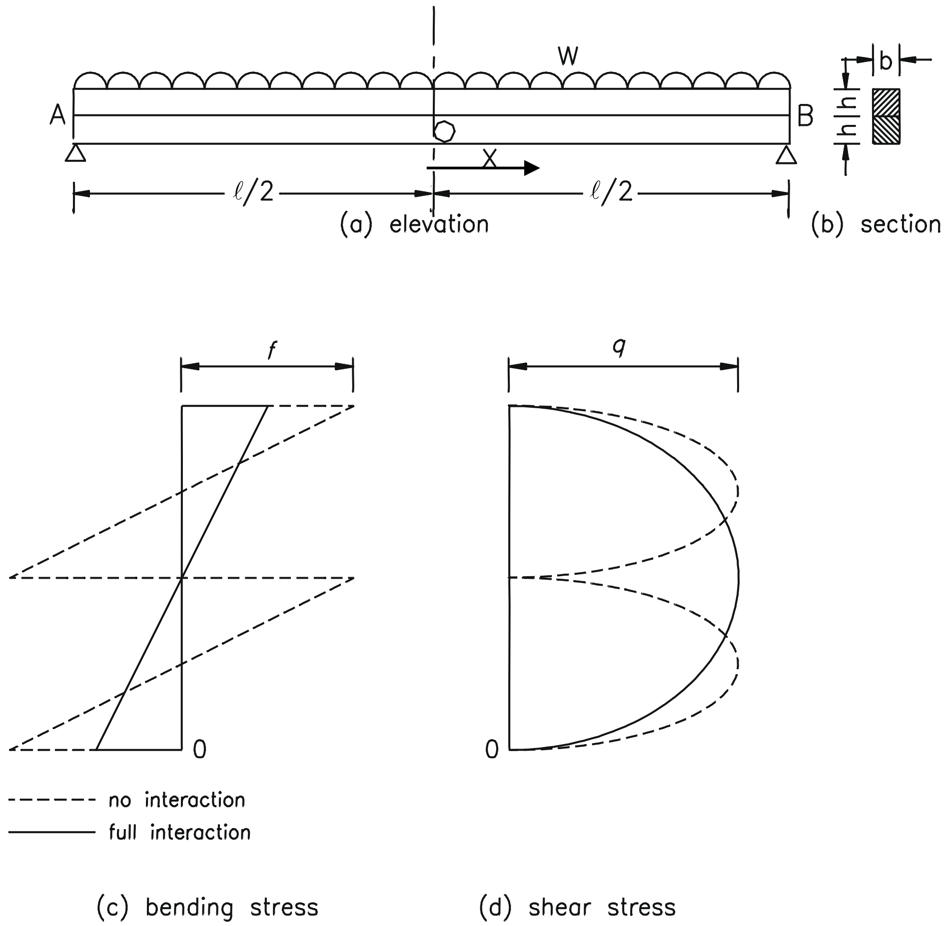


Fig. 1. Effect of shear connection on bending and shear stresses

2.1 No Interaction Case

It is first assumed that there is no shear connection between the beams, so that they are just seated on one another but act independently. The moment of inertia *(I)* of each beam is given by $bh^3/12$. The load carried by each beam is $w/2$ per unit length, with mid span moment of $w\ell^2/16$ and vertical compressive stress of $w/2b$ at the interface. From elementary beam theory, the maximum bending stress in each beam is given by,

$$f = \frac{My_{\max}}{I} = \frac{3w\ell^2}{8bh^2} \tag{1}$$

where, M is the maximum bending moment and y_{max} is the distance to the extreme fibre equal to $h/2$.

The maximum shear stress (q_{max}) *that* occurs at the neutral axis of each member near support is given by

$$q_{\max} = \frac{3}{2}\frac{w\ell}{4}\frac{1}{bh} = \frac{3w\ell}{8bh} \tag{2}$$

and the maximum deflection is given by

$$\delta = \frac{5(w/2)\ell^4}{384EI} = \frac{5w\ell^4}{64Ebh^3} \tag{3}$$

The bending moment in each beam at a distance x from mid span is,

$$M_x = w(\ell^2 - 4x^2)/16 \tag{4}$$

So, the tensile strain at the bottom fibre of the upper beam and the compression stress at the top fibre of the lower beam is,

$$\varepsilon_x = \frac{My_{max}}{EI} = \frac{3w(\ell^2 - 4x^2)}{8Ebh^2} \tag{5}$$

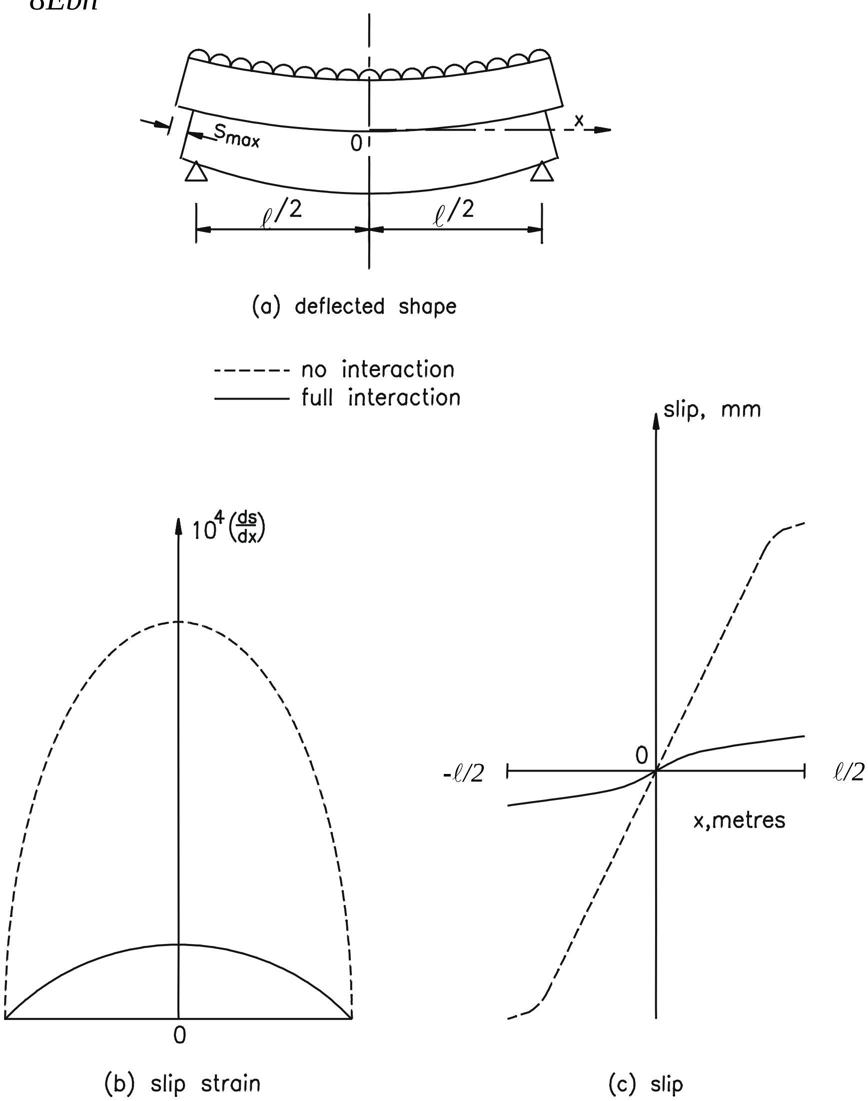


Fig. 2. Typical Deflections, slip strain and slip.

Hence the top fibre of the bottom beam undergoes slip relative to the bottom fibre of the top beam. The slip strain i.e. the relative displacement between adjacent fibres is therefore $2\varepsilon_x$. Denoting slip by *S*, we get,

$$\frac{dS}{dx} = 2\varepsilon_x = \frac{3w\,(\ell^2 - 4x^2)}{4Ebh^2} \qquad (6)$$

Integrating and applying the symmetry boundary condition $S = 0$ at $x = 0$ we get the equation

$$S = \frac{w\,(3\ell^2 x - 4x^3)}{4Ebh^2} \qquad (7)$$

The Eqn. (6) and Eqn. (7) show that at $x = 0$, slip strain is maximum whereas the slip is zero, and at $x=\ell/2$, slip is maximum whereas slip strain is zero. This is illustrated in Fig 2. The maximum slip (i.e. $S_{max} = w\ell^3/4Ebh^2$) works out to be $3.2h/\ell$ times the maximum deflection of each beam derived earlier. If $\ell/(2h)$ of beams is *20*, the slip value obtained is *0.08* times the maximum deflection. This shows that slip is a very small in comparison to deflection of beam. In order to prevent slip between the two beams at the interface and ensure bending strain compatibility shear connectors are frequently used. Since the slip at the interface is small these shear connections, for full composite action, have to be very stiff.

2.2 Full (100%) interaction case

Let us now assume that the beams are joined together by infinitely stiff shear connection along the face *AB* in Fig. 1. As slip and slip strain are now zero everywhere, this case is called "*full interaction*". In this case the depth of the composite beam is *2h* with a breadth *b*, so that $I = 2bh^3/3$. The mid-span moment is $w\ell^2/8$. The maximum bending stress is given by

$$f_{\max} = \frac{My_{\max}}{I} = \frac{w\ell^2}{8}\,\frac{3}{2bh^3}\,h = \frac{3w\ell^2}{16bh^2} \qquad (8)$$

This value is half of the bending stress given by Eqn. (1) for *"no interaction case"*. The maximum shear stress q_{max} remains unaltered but occurs at mid depth. The mid span deflection is

$$\delta = \frac{5w\ell^4}{256Ebh^3} \qquad (9)$$

This value of deflection is one fourth of that of the value obtained from Eqn. (3).

Thus by providing full shear connection between slab and beam, the strength and stiffness of the system can be significantly increased, even though the material consumption is essentially the same.

The shear stress at the interface is

$$V_x = q_x b = \frac{3wx}{4h}$$

where x is measured from the centre of the span. Fig.3 shows the variation of the shear stress. The design of the connectors has to be adequate to sustain the shear stress. In elastic design, connections are provided at varying spacing normally known as *"triangular spacing".* In this case the spacing works out to be

$$S = \frac{4Ph}{3wx}$$

where, P is the design shear resistance of a connector.

The total shear force in a half of the span is

$$V = \int_0^{\ell/2} \frac{3wx}{4h}\, dx = \frac{3w\ell^2}{32h}$$

With a value of $\ell/(2h) \approx 20$, the total shear in the whole span works out to be

$$2V = 2 \times \frac{3\,\ell}{32\,h}\, w\ell \approx 8w\ell$$ i.e. eight times the total load carried by the beam.

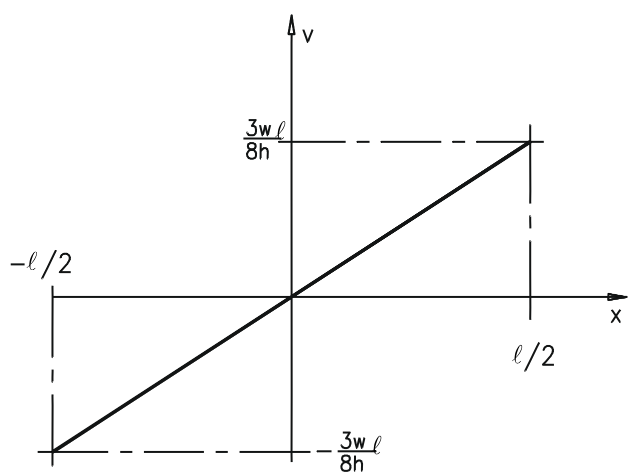


Fig.3. Shear stress variation over span length

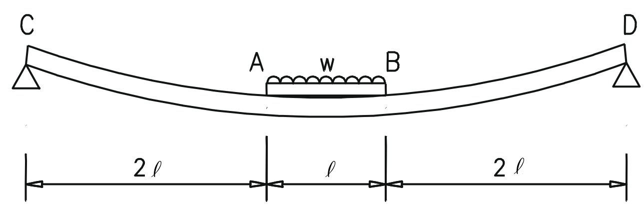


Fig. 4. Uplift forces

2.3 Uplift

Vertical separation between the members occurs, if the loading is applied at the lower edge of the beam. Besides, the torsional stiffness of reinforced concrete slab forming flanges of the composite beam and tri-axial state of stress in the vicinity of shear connector also tend to cause uplift at the interface. Consider a composite beam with partially completed flange or a non-uniform section as in Fig 4. *AB* is supported on *CD*, without any connection between them and carries a uniformly distributed load of magnitude *w*. If the flexural rigidity of *AB* is larger even by *10%* than that of *CD*, the whole load on *AB* is transferred to *CD* at *A* and *B* with a separation of the beams between these two points. If *AB* was connected to *CD*, there will be uplift forces at mid span. This shows that shear connectors are to be designed to give resistance to slip as well as uplift.

3.0 SHEAR CONNECTORS

From the previous example it is also found that the total shear force at the interface between a concrete slab and steel beam is approximately eight times the total load carried by the beam. Therefore, mechanical shear connectors are required at the steel-concrete interface. These connectors are designed to (a) transmit longitudinal shear along the interface, and (b) Prevent separation of steel beam and concrete slab at the interface.

3.1 Types of shear connectors

3.1.1 Rigid type

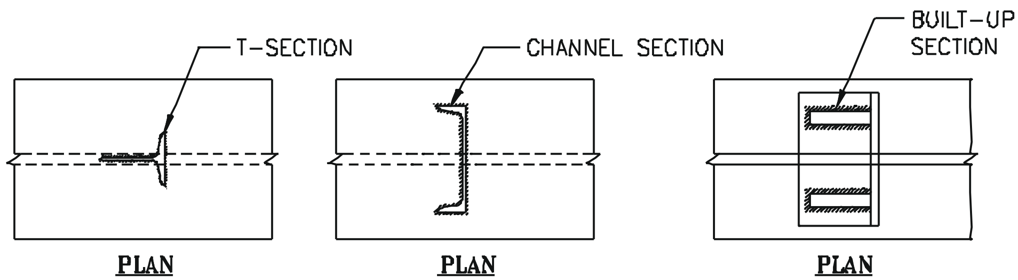
As the name implies, these connectors are very stiff and they sustain only a small deformation while resisting the shear force. They derive their resistance from bearing pressure on the concrete, and fail due to crushing of concrete. Short bars, angles, T-sections are common examples of this type of connectors. Also anchorage devices like hooped bars are attached with these connectors to prevent vertical separation. This type of connectors is shown in Fig 5(*a*).

3.1.2 Flexible type

Headed studs, channels come under this category. These connectors are welded to the flange of the steel beam. They derive their stress resistance through bending and undergo large deformation before failure. Typical flexible connectors are shown in Fig 5(*b*). The stud connectors are the types used extensively. The shank and the weld collar adjacent to steel beam resist the shear loads whereas the head resists the uplift.

3.1.3 Bond or anchorage type

These connectors derive their resistance through bond and anchorage action. These are shown in Fig 5(*c*). The dimensions of typical shear connectors as per IS: 11384 –1985 are given in Fig 6.



M.S. bars threaded through holes in the shear connectors

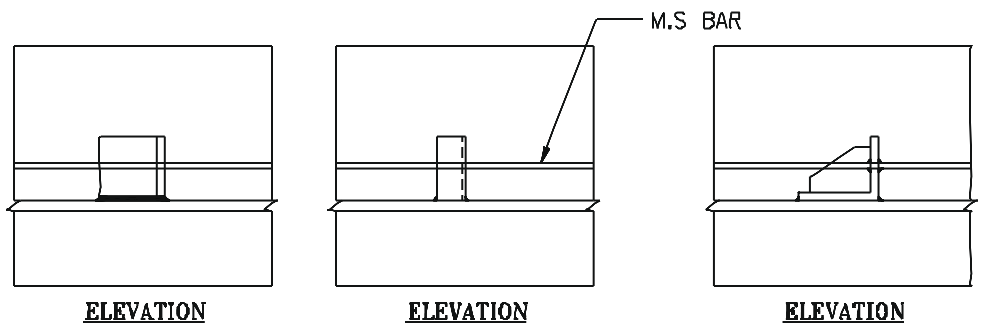


Fig. 5(a). Typical rigid connectors with anchorage device to hold down the concrete slab against uplift

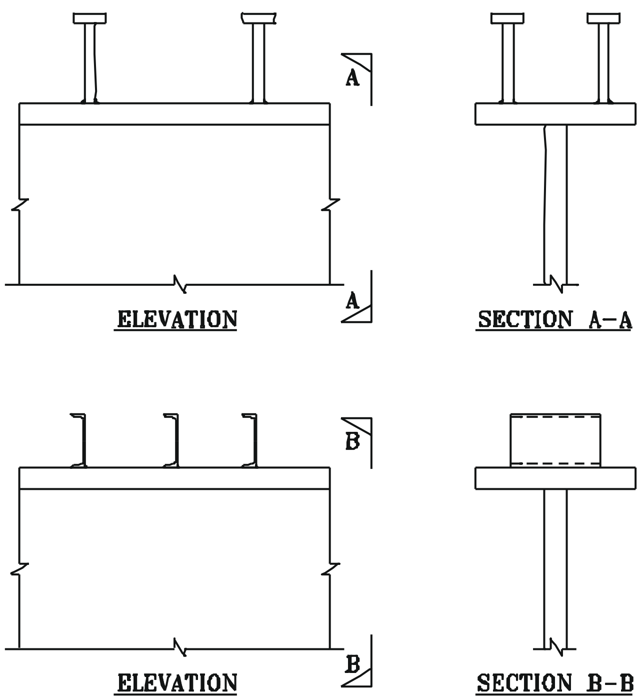
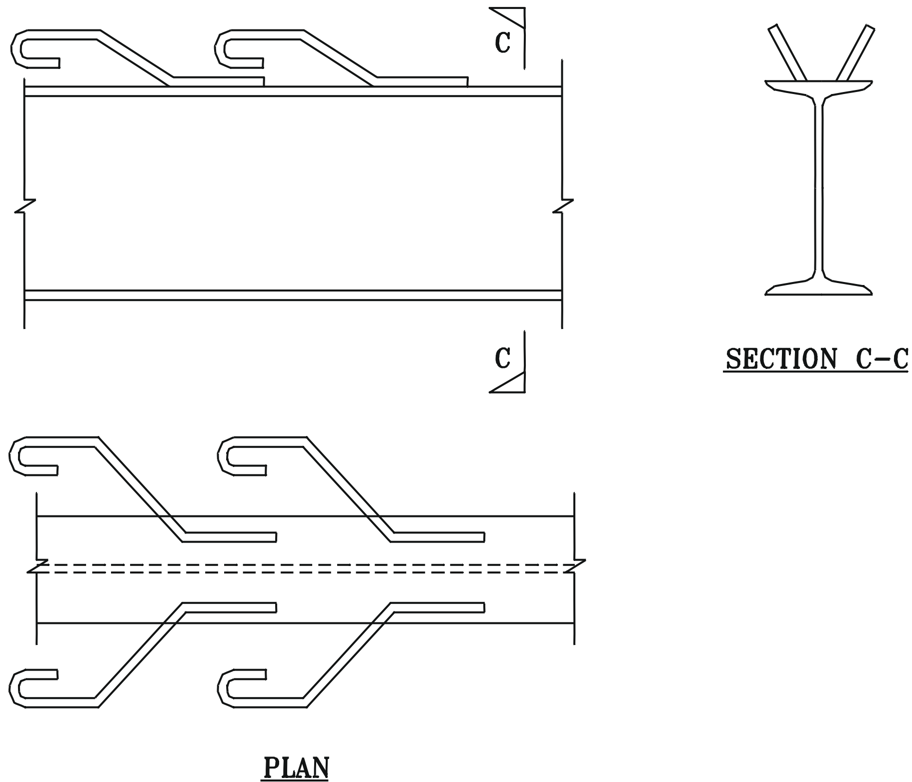
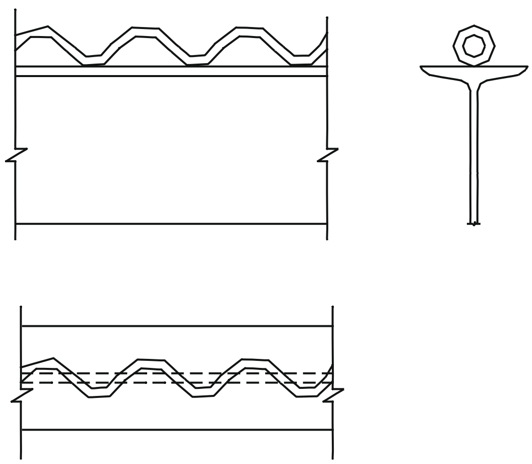


Fig. 5(b). Typical flexible connectors

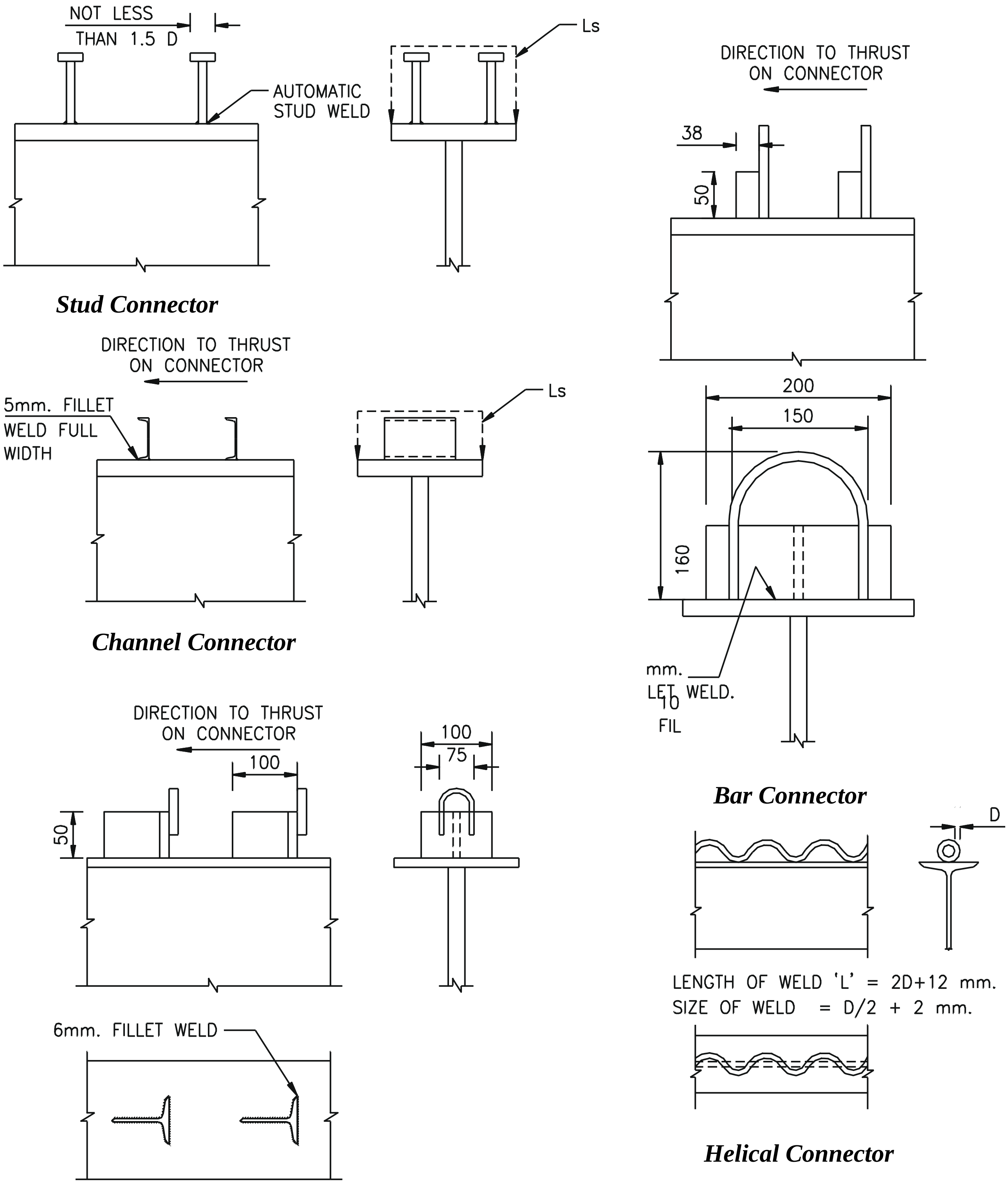


(i). Inclined mild steel bars welded to the top flange of steel unit



(ii). Helical connector

Fig. 5(c). Typical bond or anchorage connectors



Tee Connector *All dimensions as per IS 11384: 1985*

Fig. 5(d) Typical shear connectors

Fig. 5. Typical rigid connectors with anchorage device to hold down the concrete slab against uplift

3.2 Characteristics of shear connectors

Though in the discussion of full interaction it was assumed that slip was zero everywhere, results of tests have proved that even at the smallest load, slip occurs. This load-slip characteristic of shear connectors affects the design considerably. To obtain the load-slip curve "***push- out***" tests are performed. Arrangements for these tests as per Eurocode 4 and IS: 11384-1985 are shown in Fig. 6(*a*) and 6(*b*) respectively.

In "push-out" test two small slabs are connected to the flanges of an *I* section. The slabs are bedded onto the lower platen of a compression-testing machine and load is applied on the upper end of the steel section. A load-slip curve is obtained by plotting the average slip against the load per connector.

To perform the test, IS:11384-1985 suggests that,

- at the time of testing, the characteristic strength of concrete used n_p / n_f should not exceed the characteristic strength of concrete in the beams for which the test is designed.
- a minimum of three tests should be made and the design values should be taken as *67%* of the lowest ultimate capacity.

Fig 7(*a*) shows trend of some of the results of "***push -out***" tests on different shear connectors. The brittle connectors reach their peak resistance with relatively small slip and then fail suddenly, but the ductile connectors maintain their shear carrying capacity over large displacements. Based on the load slip curve two important parameters can be obtained- the plastic plateau and the connector stiffness *k*. While ultimate strength analysis is based on plastic behaviour of shear connectors, the *'k'* value is required for serviceability analysis and to find slip strain and stresses at partial interaction. In the ultimate analysis it is assumed that concrete slab, steel beam and the dowel are fully stressed, which is known as *"rigid plastic"* condition. In this condition the flexural strength of the section is determined from equilibrium equation. This can be seen from the idealised load-slip characteristics of connectors as in Fig 7(*c*).

Fig 7(*c*) shows an idealised load-slip characteristic of three different types of interaction that can arise depending on the type of connectors used. Note that full interaction occurs when $k=\infty$ represented by an arrow along Y-axis. This occurs when very stiff connectors are used. When there is partial interaction the load slip relationship is assumed to be bilinear. The ultimate capacity is reached at a shear load of D_{max} and only, thereafter slip occurs even without increase in shear load. The stiffness *k* for this partial interaction is assumed to be constant from zero shear loads up to D_{max}.

The dotted lines in the Fig 7(*c*) show the rigid plastic interaction with finite slip (S_{ult}) behaviour. This has a definite plastic plateau and assume $k=\infty$ at occurring at the maximum shear load indicated by D_{max}.

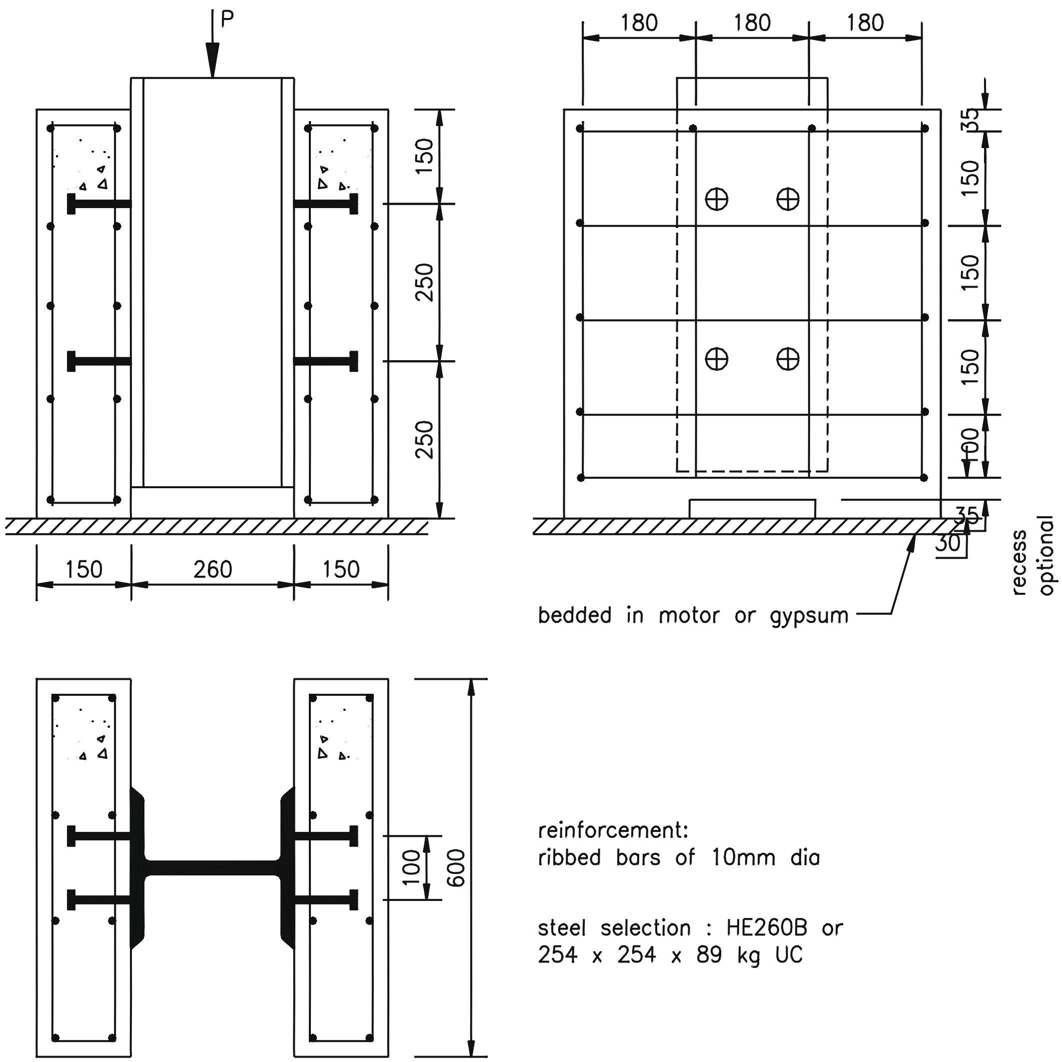


Fig. 6(a). Standard push test *[Eurocode – 4]*

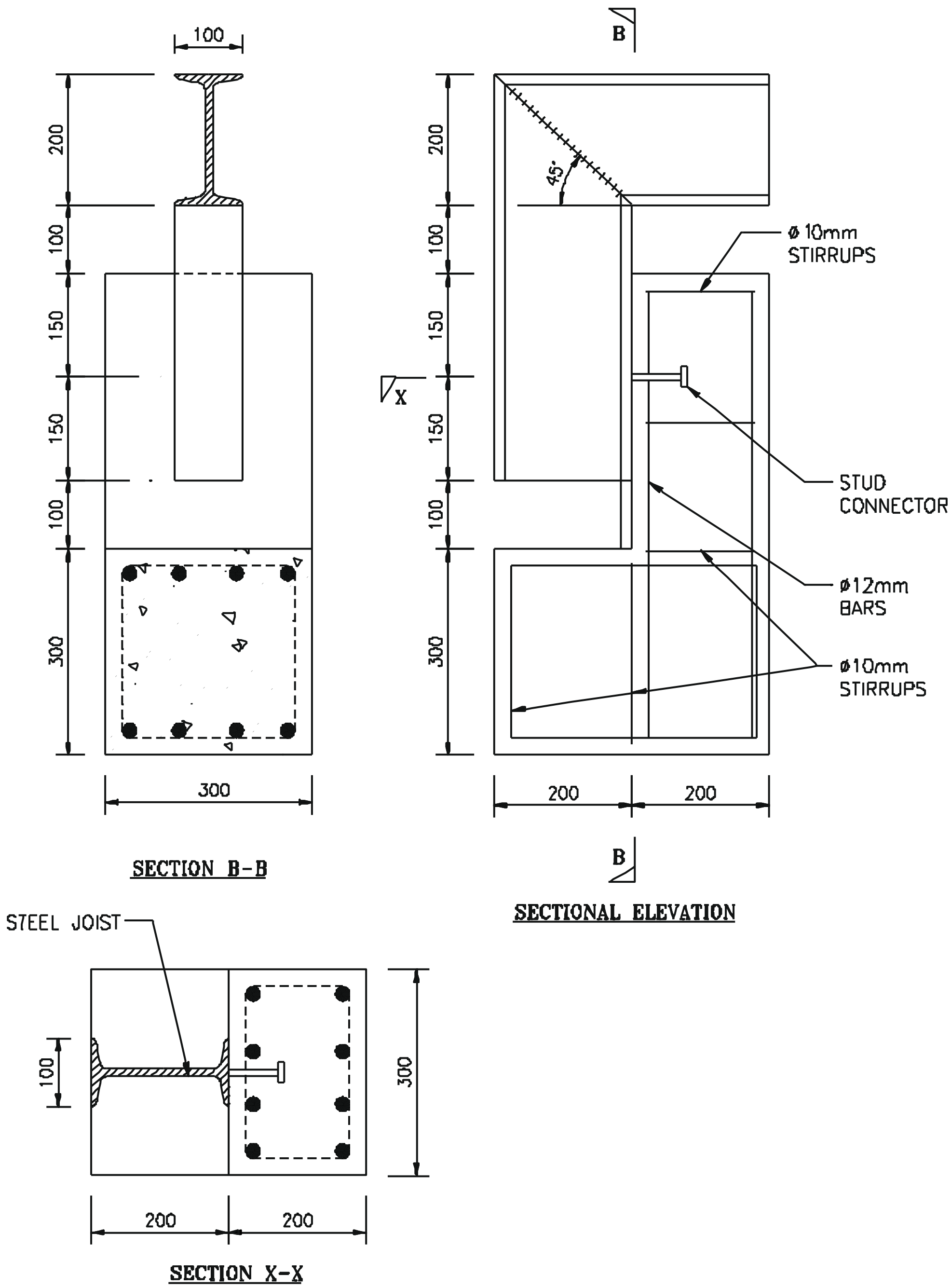


Fig. 6(b) Standard test for shear connectors *(As per IS: 11384-1985)*

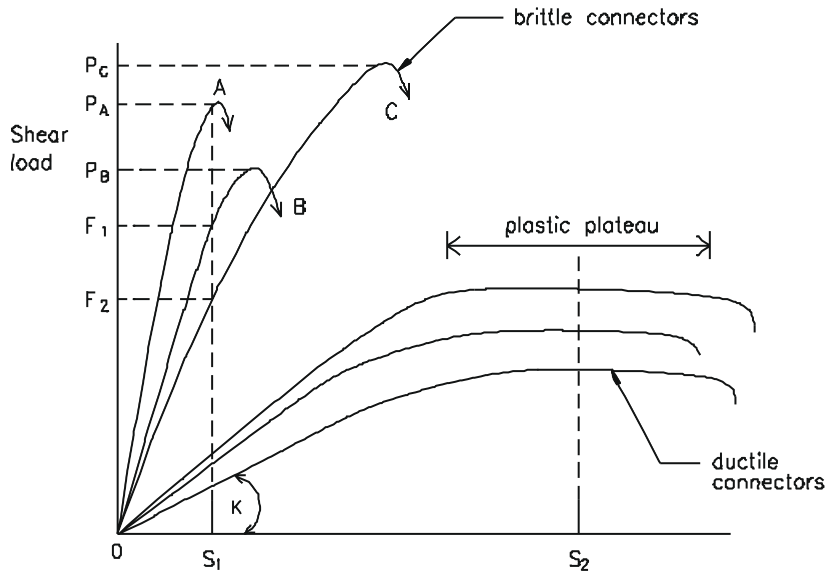


Fig. 7(a). Load/Slip characteristics

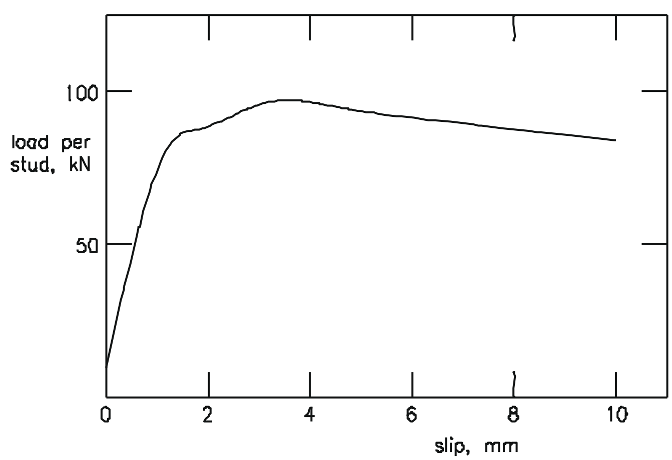


Fig. 7(b). Typical load-slip curve for 19mm stud connectors

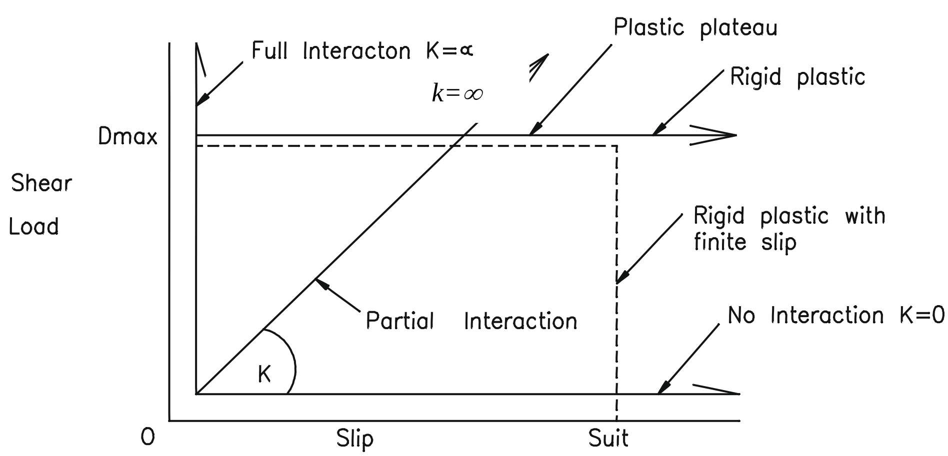


Fig. 7(c). Idealized load-slip characteristics

3.3 Load bearing mechanism of shear connectors

In the course of resisting the shear load, the connectors deform and transfer the load to concrete through bearing. This is illustrated in Fig 8. The dispersion of load can cause tensile cracks in concrete by ripping, shear and splitting action, shown in Fig 9. However the steel dowel may also fail before concrete fails.

Though the transfer of longitudinal shear through mechanical shear connectors is a very complex mechanism, it is shown in an idealised manner in Fig 10. Here the resultant force F acts at an eccentricity *'e'* from the interface. It has been found by research that the bearing stress on a shank is concentrated near the base as in Fig 11. Assuming that the force is distributed over a length of *2d* where *d* is the shank diameter, it can be shown that concrete has to withstand a bearing stress of about five times its cube strength. This high strength is possible, because concrete bearing on the connector is confined laterally by the steel element, reinforcement and surrounding concrete. Referring to Fig. 10, we find that, for equilibrium, horizontal shear force as well as a moment is induced at the base of the connector. So, the steel dowel must resist shear as well as flexural forces that cause high tensile stress in the steel failure zone.

However, in a better approach, considering the frictional force between the dowel and the concrete, it has been found that eccentricity *'e'* depends upon E_s/E_c also. As E_s increases, the bearing pressure on dowel becomes more uniform, increasing the eccentricity *'e'*. As a result, flexural force $F.e$ increases reducing the dowel strength. On the other hand with decrease in E_s, dowel strength increases.

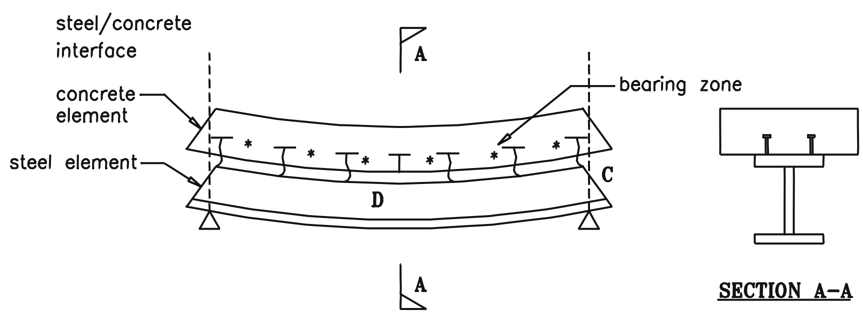


Fig.8 Load Bearing Mechanism

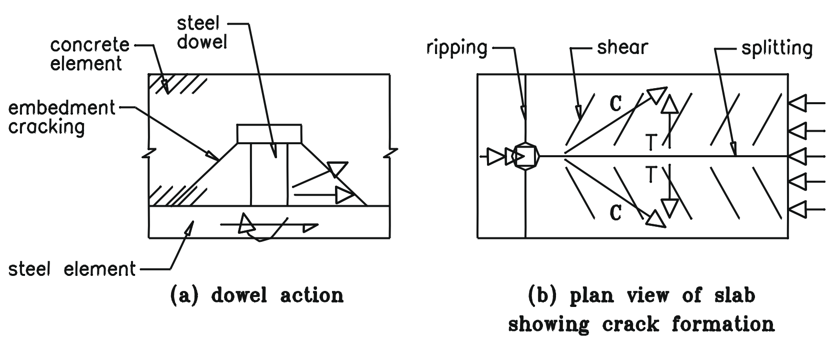


Fig. 11 Bearing stress on the shank of a stud connector

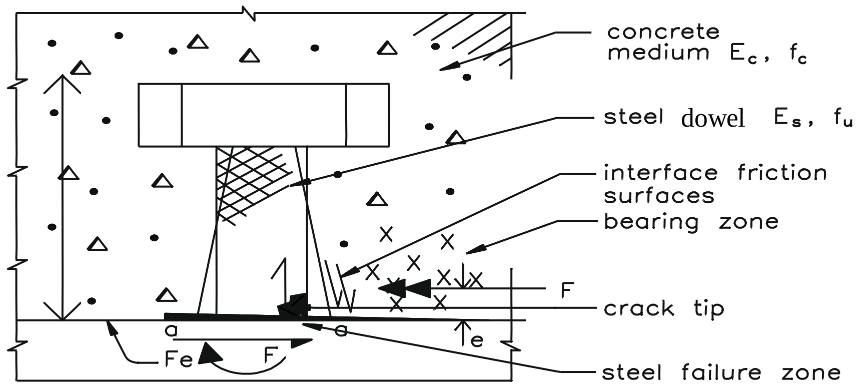


Fig.10. Dowel mechanism of shear studs

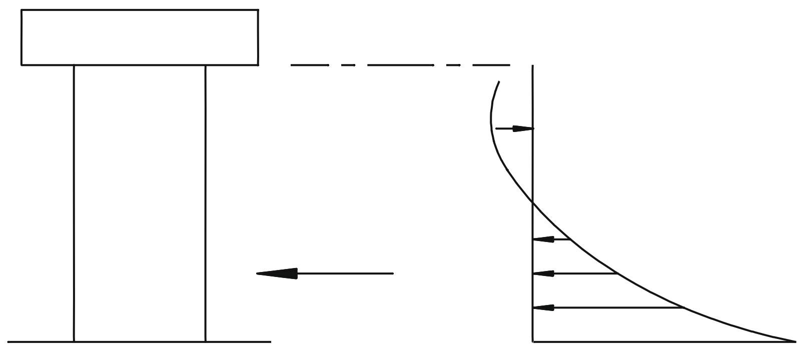


Fig. 11 Bearing stress on the shank of a stud connecto

3.4 Strength of connectors

From the above discussion, it can be inferred that dowel strength *(D)* is a function of the following parameters: -

$D = f\ [A_d, f_u, (f_{ck})_{cy}, E_c/E_s]$

where

A_d = cross section area of dowel

f_u = tensile strength of steel

$(f_{ck})_{cy}$ =characteristic compressive (cylinder) strength of concrete

E_c/E_s = ratio of modulus of elasticity of concrete to that of steel.

Table 1: Design Strength of Shear Connectors for Different Concrete Strengths

Type of Connectors		**Connector Material**	**Design Strength of Connectors for Concrete of Grade**		
			M-20	**M-30**	**M-40**
I. Headed stud		IS:961-1975* Fe 540-HT	Load per stud (P_c), kN		
Diameter mm	Height mm				
25	100		86	101	113
22	100		70	85	94
20	100		57	68	75
20	75		49	58	64
16	75		47	49	54
12	62		23	28	31
II. Bar Connector		IS:2261975	Load per bar KN		
50mm x 38 mm x 200mm			318	477	645
III. Channel connector		IS226-1975	Load per channel	(P_c)kN	
125mm x 65mm x 12.7kg x 150mm			184	219	243
100mm x 50mm x 9.2kg x 150 mm			169	204	228
75mm x 40mm x 6.8 kg x 150mm			159	193	218
IV. Tee connector		IS:226-1975	Load per connector (P_c)kN		
100 mm x 100 mm x 10 mm Tee x 50mm			163	193	211
V. Helical connector		IS:226-1975	Load per pitch	(P_c)kN	
Bar diameter mm	Pitch circle diameter mm	IS:226-1975	Load per pitch	(P_c)kN	
20	125		131	154	167
16	125		100	118	96
12	100		70	83	90
10	75		40	48	52

A typical load slip curve for 19 mm-stud connector is shown in Fig. 7(*b*). Eurocode 4 has given the following two empirical formulae to find design resistance of shear studs with $h/d \geq 4$.

$$P_{Rd} = \frac{0.8 f_u (\pi d^2 / 4)}{\gamma_v} \qquad (10)$$

$$P_{Rd} = \frac{0.29\, d^2\, ((f_{ck})_{cy} E_{cm})^{1/2}}{\gamma_v} \qquad (11)$$

f_u = ultimate tensile strength of steel *($\le$500 N/mm^2)*
$(f_{ck})_{cy}$ = cylinder strength of concrete
E_{cm} = mean secant (elastic) modulus of concrete.
γ_v = partial safety factor for stud connector = *1.25*

Equation. (*10*) is based on failure of the shank whereas Equation. (*11*) is based on failure in concrete. The lower of the above two values governs the design.

The design strength of some commonly used shear connectors as per IS:11384-1985 is given in Table (1).

It is to be noted that as per this code the design value of a shear connector is taken as *67%* of the ultimate capacity arrived at by testing.

4.0 ULTIMATE LOAD BEHAVIOUR OF COMPOSITE BEAM

The design procedure of composite beams depends upon the class of the compression flange and web. Table (2) shows the classification of the sections suggested in Eurocode 4 based upon the buckling tendency of steel flange or web. The resistance to buckling is a function of width to thickness ratio of compression members. Table (2) shows that for sections falling in Class 1 & 2 (in EC 4, See Table 2), plastic analysis is recommended. For simply supported composite beams the steel compression flange is restrained from local as well as lateral buckling due to its connection to concrete slab. Moreover, the plastic neutral axis is usually within the slab or the steel flange for full interaction. So, the web is not in compression. This allows the composite section to be analysed using plastic method. Results obtained from plastic analysis have been found to be in close agreement with those obtained from test.

Table 2: Classification of sections, and methods of analysis (according to Eurocode4)

Slenderness class and name	1 plastic	2 compact	3 semi-compact	4 slender
Method of global analysis Analysis of cross-sections	plastic$^{(4)}$ plastic$^{(4)}$	elastic plastic$^{(4)}$	elastic elastic$^{(1)}$	elastic elastic$^{(2)}$
Maximum ratio of *c/t* for flanges of rolled *I*-section: (3)				
Uncased web	8.14	8.95	12.2	no limit
Encased web	8.14	12.2	17.1	no limit

Notes: (1) hole-in-the-web method enables plastic analysis to be used:
(2) with reduced effective width or yield strength

(3) for Grade *50* steel (f_y = *355 N/mm²*): *c* is half the width of a flange of thickness *t;*

(4) Elastic analysis may be used, but is more conservative.

The *assumptions* made for the analyses of the Ultimate Moment Capacity of the section (according to Eurocode 4) are as follows: -

- The tensile strength of concrete is ignored.
- Plane sections of both structural steel and reinforced concrete remain plane after bending.
- The effective area of concrete resists a constant stress of $0.85\ (f_{ck})_{cy}/\gamma_c$ (where $(f_{ck})_{cy}$)=cylinder strength of concrete; and γ_c =partial safety factor for concrete) over the depth between plastic neutral axis and the most compressed fibre of concrete.
- The effective area of steel member is stressed to its design yield strength f_y/γ_a where f_y is the yield strength of steel and γ_a is the material safety factor for steel.

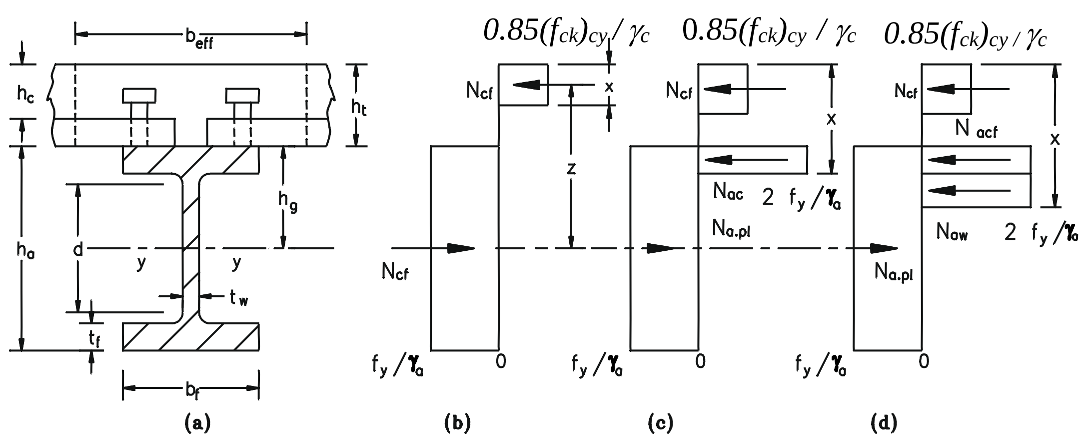


Fig.12. Resistance to sagging bending of composite section in class 1 or 2 for full interaction.

The notations used here are as follows: -

A_a =area of steel section
γ_a = partial safety factor for structural steel
γ_c = partial safety factor for concrete
b_{eff} =effective width of flange of slab
f_y =yield strength of steel
$(f_{ck})_{cy}$ =characteristic (cylinder) compressive strength of concrete
h_c =distance of rib from top of concrete
h_t =total depth of concrete slab
h_g =depth of centre of steel section from top of steel flange

Note: Cylinder strength of concrete $(f_{ck})_{cy}$ is usually taken as *0.8* times the cube strength.

4.1 Full shear connection

Assuming full interaction following three cases may arise.

1) Neutral axis within the concrete slab [see Fig. 12(*b*)].

This occurs when

$$0.85\,\frac{(f_{ck})_{cy}}{\gamma_c}\,b_{eff}\,h_c \geq \frac{A_a f_y}{\gamma_a} \qquad (12)$$

The depth of plastic neutral axis can be found by using force equilibrium.

$$N_{cf} = \frac{A_a f_y}{\gamma_a} = b_{eff}\;x\,\frac{0.85(f_{ck})_{cy}}{\gamma_c} \qquad (13)$$

$$\therefore \quad x = \frac{\dfrac{A_a f_y}{\gamma_a}}{\dfrac{0.85(f_{ck})_{cy}}{\gamma_c}\,b_{eff}} \qquad (14)$$

This expression is valid for $x \leq h_c$.

The plastic moment of resistance of the section,

$$M_p = \frac{A_a f_y}{\gamma_a}\,(h_g + h_t - x/2) \qquad (15)$$

2) Neutral axis within the steel top flange [see Fig.12(*c*)]

This case arises when

$N_{cf} < N_{a.pl}$

$$i.e. \quad b_{eff}\,h_c\,\frac{0.85(f_{ck})_{cy}}{\gamma_c} < \frac{A_a\,f_y}{\gamma_a} \qquad (16)$$

To simplify the calculation it is assumed that strength of steel in compression is $2\,f_y/\gamma_a$, so that, the force $N_{a.pl}$ and its line of action remain unchanged. Note that the compression flange is assumed to have a tensile stress of f_y/γ_a and a compressive stress of $2f_y/\gamma_a$, giving a net compressive stress of f_y/γ_a.

So, the plastic neutral axis will be within steel flange if

$N_{a.pl} - N_{cf} \leq \ 2\, b_f\, t_f\, f_y/\gamma_a$

Equating tensile force with compressive,

$N_{a.pl} = N_{cf} + N_{ac}$

i.e. $$\frac{A_a f_y}{\gamma_a} = \frac{0.85(f_{ck})_{cy}}{\gamma_c} b_{eff} h_c + 2b_f\,(x - h_t)\frac{f_y}{\gamma_a} \qquad (17)$$

The value of x is found from the above expression.
The plastic moment of resistance is found from

$$M_p = N_{a.pl}\,(h_g + h_t - h_c/2) - N_{ac}\,(x - h_c + h_t)/2 \qquad (18)$$

3) The neutral axis lies within web (see Figure 12d).

If the value of x exceeds $(h_c + t_f)$, then the neutral axis lies in the web. In design this case should be avoided, otherwise the web has to be checked for slenderness.

In similar procedure as the previous one, here x can be found from

$$\begin{aligned} N_{a.pl} &= N_{cf} + N_{acf} + N_{aw} \\ &= N_{cf} + 2b_f t_f\, f_y/\gamma_a + 2t_w\,(x - h_t - t_f)\, f_y/\gamma_a \end{aligned} \qquad (19)$$

Plastic moment of resistance

$$\begin{aligned} M_p = N_{a.pl}\,(h_g + h_t - h_c/2) &- N_{acf}\,(h_t + t_f/2 - h_c/2) \\ &- N_{a.w}(x + h_t + t_f - h_c)/2 \end{aligned} \qquad (20)$$

4.2 Partial shear connection

Sometimes due to the problem of accommodating shear connectors uniformly or to achieve economy, partial shear connections are provided between steel beam and concrete slab. Then the force resisted by the connectors are taken as their total capacity $(F_c < F_{cf})$ between points of zero and maximum moment. Here F_{cf} refers to the term of N_{cf} as used earlier. If n_f and n_p are the number of shear connectors required for full interaction and partial interaction respectively, then the degree of shear connection is defined as, n_p / n_f. Therefore,

$$\textit{Degree of shear connection} = \frac{n_p}{n_f} = \frac{F_c}{F_{cf}}$$

Assuming all connectors have same resistance to shear, The depth of compressive stress block in slab is

$$x_c = \frac{F_c}{0.85b(f_{ck})_{cy}/\gamma_c} \tag{21}$$

which is less than h_c.

The neutral axis of steel section may be in the flange or in the web. In case the neutral axis is within top flange, the moment of resistance can be found out using stress block shown in Figure *12c.* Here the block of N_{cf} is replaced by F_c therefore,

$$M_{Rd} = N_{a.pl}(h_g + h_t - \frac{x_c}{2}) - F_c\,\frac{x_a + h_t - x_c}{2} \tag{22}$$

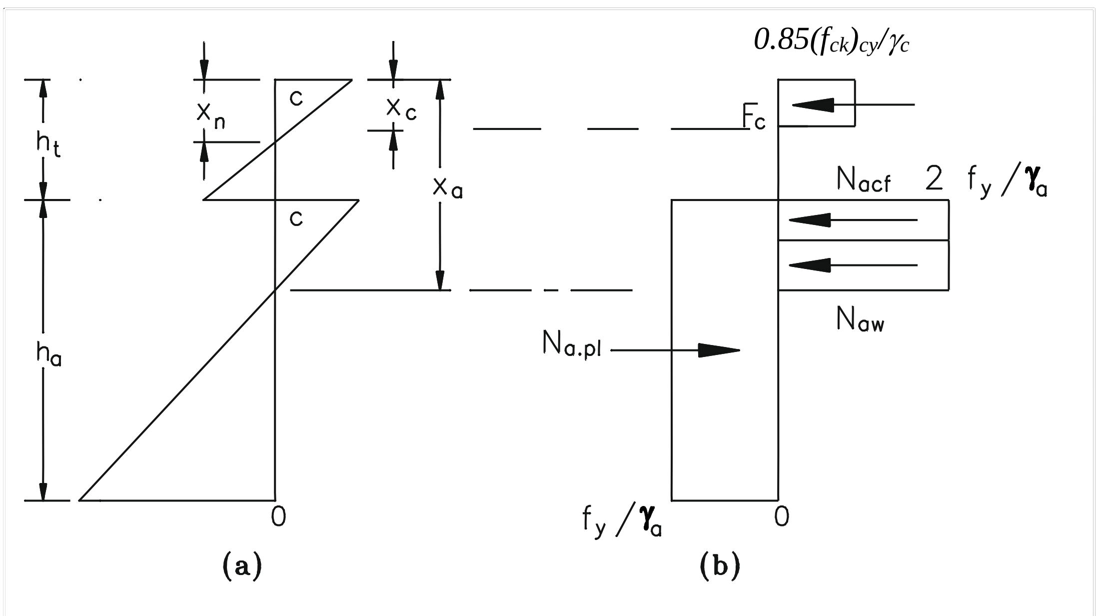


Fig.13. Resistance to sagging bending of composite section in class 1 or 2 for partial interaction

If the neutral axis lies in web (refer Fig. 13) the moment of resistance is determined by taking moment about top surface

$$M_{Rd} = N_{a.pl}(h_g + h_t) - \frac{F_c x_c}{2} - N_{cf}(h_t + \frac{t_f}{2}) - N_{aw}\,\frac{x_a + h_t - t_f}{2} \tag{23}$$

where $N_{acf} = 2b_f\, t_f\, \dfrac{f_y}{\gamma_a}$

$N_{aw} = N_{a.pl} - F_c$ - N_{acf}

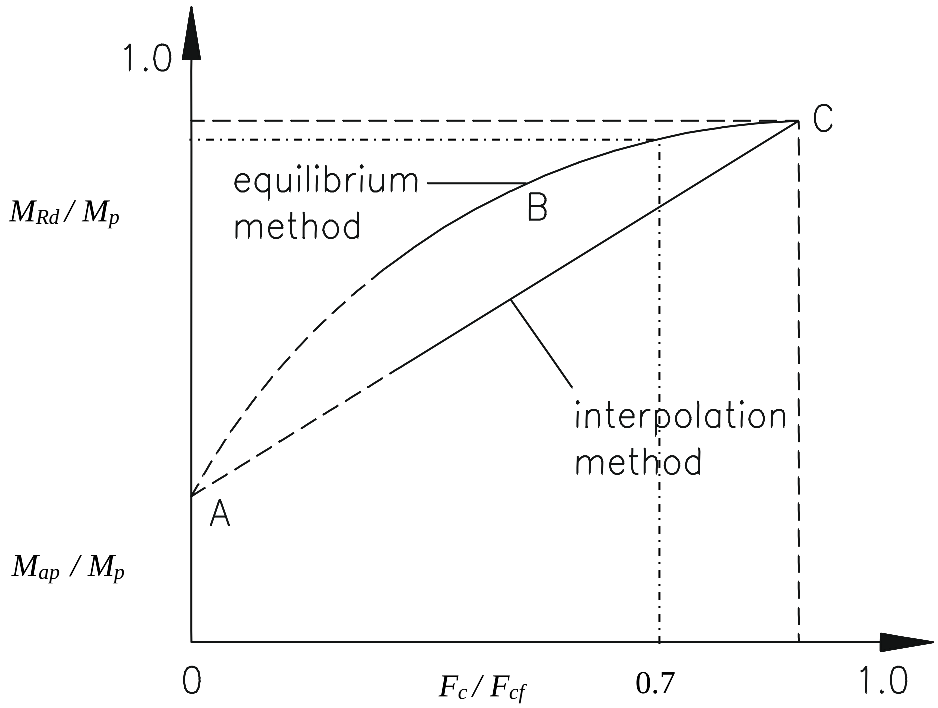


Fig.14. Design methods of partial shear connection

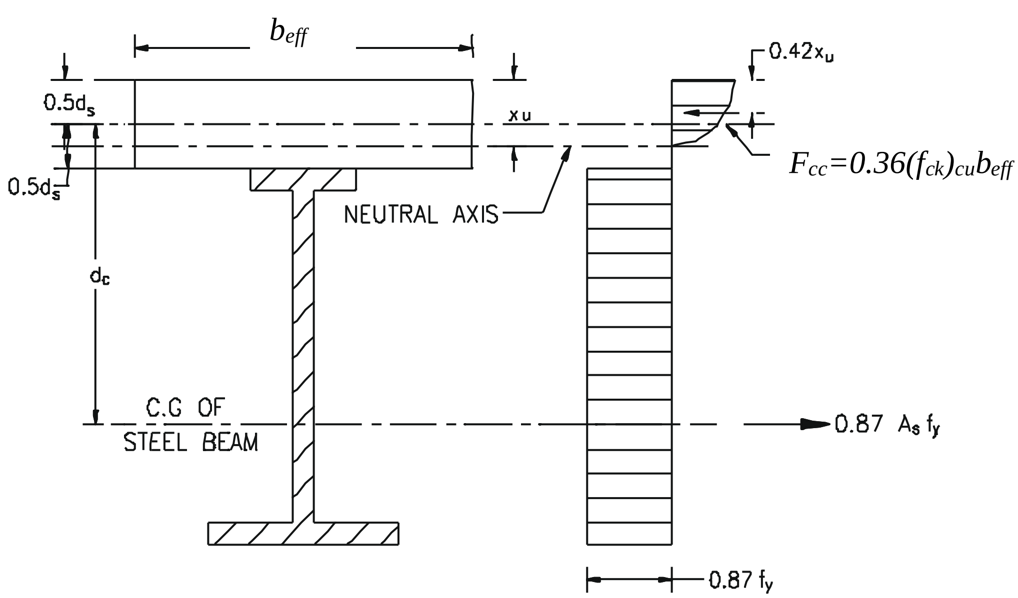


Fig 15 Stress distribution in a composite beam with neutral axis within concrete slab

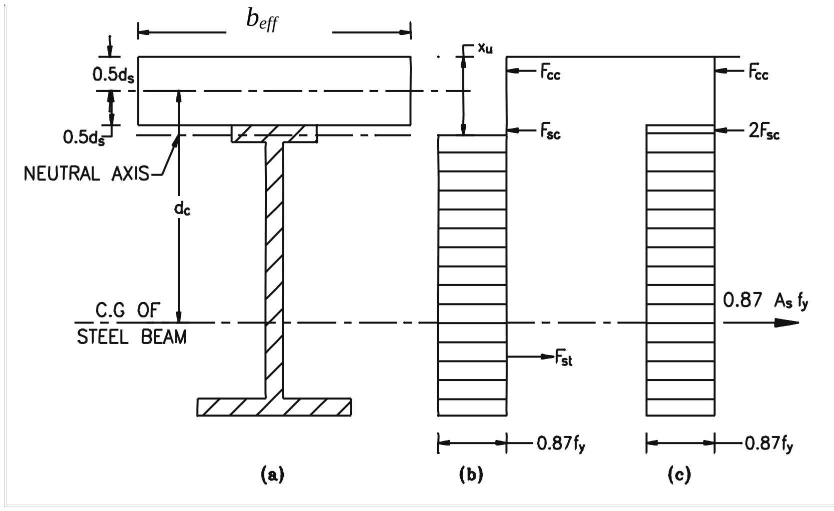


Fig.16. Stress distribution in a composite beam with neutral axis with in flange of steel beam.

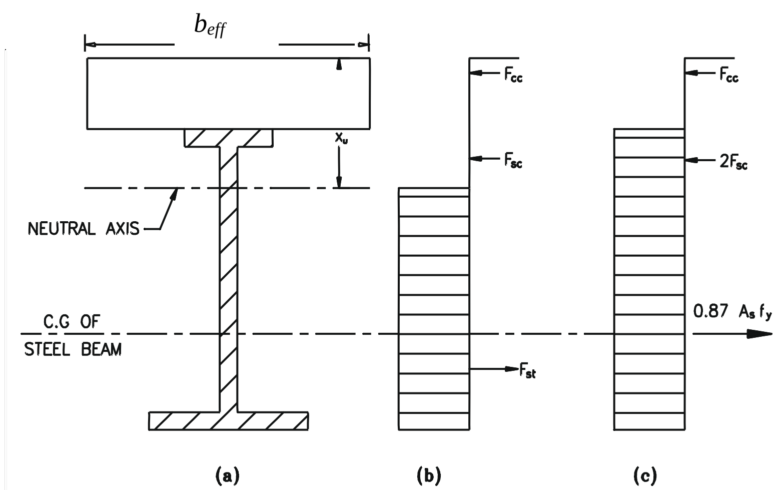


Fig.17 Stress distribution in a composite beam with neutral axis within the web of the steel beam.

Moment of resistance reduces due to partial shear connection. The relation between M_{Rd}/M_p with degree of shear connection F_c/F_{cf} is shown in Fig.14. The curve *ABC* is not valid for very low value of shear connection. It can be seen from the curve that at $F_c/F_{cf} = 0.7$, the required bending resistance is slightly below M_p. Using this a considerable saving in the cost of shear connectors can be achieved without unduly sacrificing the moment capacity. However for design purpose the curve *ABC* is replaced by a straight-line *AC* given by

$$S_c = \frac{M - M_{ap}}{M_p - M_{ap}} F_{cf} \tag{24}$$

where, M is the required bending resistance of the section; and

M_{ap} is the plastic moment of resistance of steel section only.

Using the values of M_{sd}, M_{ap}, M_p and F_c, the values of F_{cf} can be found. With the value of F_{cf}, number of shear connectors per span is determined.

However, the Indian Code of Practice for Composite Construction in Structural Steel and Concrete (IS: 11384 – 1985), has adopted a parabolic stress block in concrete slab for plastic analysis of the section. Here a stress factor $a = 0.87\, f_y/0.36(f_{ck})_{cu}$ is applied to convert the concrete section into steel. The additional assumptions made by IS: 11384 – 1985 (compared to those of Eurocode) are given below:

- The maximum strain in concrete at outermost compression member is taken as 0.0035 in bending.
- The total compressive force in concrete is given by $F_{cc} = 0.36\,(f_{ck})_{cu}\, bx_u$ and this acts at a depth of $0.42\, x_u$, not exceeding d_s.
- The stress strain curve for steel section and concrete are as per IS: 456-1978.

The notations used here are

A_f = area of top flange of steel beam of a composite section.
A_s = cross sectional area of steel beam of a composite section.
b_{eff} = effective width of concrete slab.
b_f = width of top flange of steel section.
d_c = vertical distance between centroids of concrete slabs and steel beam in a composite section.
t_f = average thickness of the top flange of the steel section.
x_u = depth of neutral axis at ultimate limit state of flexure
M_u = ultimate bending moment.

The three cases that may arise are given below with corresponding M_u.

Case I: Plastic neutral axis within the slab (Refer Fig.15)

This occurs when $b_{eff}\, d_s \geq a\, A_s$.

Taking moment about centre of concrete compression

$$M_u = 0.87 A_s f_y (d_c + 0.5 d_s - 0.42 x_u) \qquad (25)$$

where,

$x_u = aA_s/b_{eff}$

$$a = \frac{0.87 f_y}{0.36 (f_{ck})_{cu}}$$

Case II: Plastic neutral axis within the top flange of steel section (Refer Fig. 16).

This happens when

$b_{eff}\, d_s < aA_s < (b_{eff}\, d_s + 2a\, A_f)$

Equating forces as in Eqn. (17) we get

$$x_u = d_s + \frac{aA_s - b_{eff}\ d_s}{2b_f\, a} \tag{26}$$

Taking moment about centre of concrete compression.

$$M_u = 0.87\, f_y \left\lfloor A_s (d_c + 0.08 d_s) - b_f (x_u - d_s)(x_u + 0.16 d_s) \right\rfloor \tag{27}$$

Case III: Plastic neutral axis lies within web (Refer Fig. 17.) This occurs when

$a\ (A_s - 2A_f) > b_{eff}\, d_s$

Equating area under tension and compression as in Equation. (19) we get

$$x_u = d_s + t_f + \frac{a(A_s - 2A_f) - b_{eff}\, d_s}{2at_w} \tag{28}$$

Taking moment about the centre of concrete compression

$$\begin{aligned} M_u = {} & 0.87 f_y A_s (d_c + 0.08 d_s) - 2A_f (0.5 t_f + 0.58 d_s) \\ & - 2t_w (x_u - d_s - t_f)(0.5\, x_u + 0.08\, d_s + 0.5 t_f) \end{aligned} \tag{29}$$

Note: ***In IS: 11384 – 1985 no reference has been made to profiled deck slab and partial shear connection. Therefore the above equations can be used only for composite beams without profiled deck sheeting (i.e., steel beam supporting concrete slabs).***

5.0 SERVICEABILITY LIMIT STATES

For simply supported composite beams the most critical serviceability Limit State is usually deflection. This would be a governing factor in design for un-propped construction. Besides, the effect of vibration, cracking of concrete, etc. should also be checked under serviceability criteria. Often in exposed condition, it is preferable to design to obtain full slab in compression to avoid cracking in the shear connector region.

5.1 Stresses and deflection in service

As structural steel is supposed not to yield at service load, elastic analysis is employed in establishing the serviceability performance of composite beam. In this method the concrete area is converted into equivalent steel area by applying modular ratio $m = (E_s/E_c)$. The analysis is done in terms of equivalent steel section. It is assumed that full interaction exists between steel beam and concrete slab. The effect of reinforcement in compression, the concrete in tension and the concrete between rib of profiled sheeting are ignored.

Refer to Fig.18, where a transformed section is shown.

When neutral axis lies within the slab

$$A_a (Z_g - h_c) < \tfrac{1}{2} b_{eff} \frac{h_c^2}{m} \tag{30}$$

The actual neutral axis depth can be found from

$$A_a (Z_g - x) = \tfrac{1}{2} b_{eff} \frac{x^2}{m} \tag{31}$$

and the moment of inertia of the transformed section is

$$I = I_a + A_a (Z_g - x)^2 + \left(\frac{b_{eff}}{m}\right) \frac{x^3}{3} \tag{32}$$

When neutral axis depth exceeds h_c, its depth x is found from the following equation.

$$A_a (Z_g - x) = \frac{b_{eff}}{m} h_c \left(x - \frac{h_c}{2} \right) \tag{33}$$

and moment of inertia of the transformed section

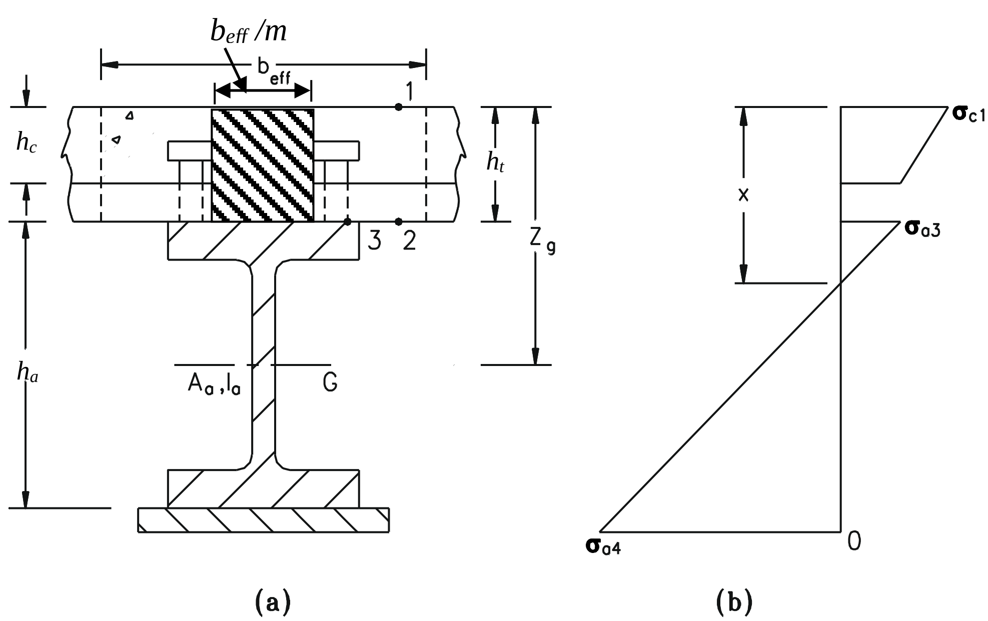


Fig.18. Elastic analysis of composite beam section in sagging bending

$$I = I_a + A_a\left(Z_g - x\right)^2 + \frac{b_{eff}\,h_c}{m}\left(\frac{h_c^{\,2}}{12} + \left(x - {h_c}/{2}\right)^2\right) \qquad (34)$$

For distributed load *w* over a simply supported composite beam, the deflection at mid-span is

$$\delta_c = \frac{5wL^4}{384E_aI} \qquad (35)$$

where E_a = Young's Modulus for structural steel.
I = moment of inertia found from Equation. (32) and Equation. (34) as applicable.

The beam can be checked for stresses under service load using the value of *'I'* as determined above.

When the shear connections is only partial the increase in deflection occurs due to longitudinal slip. This depends on method of construction. Total deflection,

$$\delta = \delta_c\left(1 + k\left(1 - \frac{N}{N_f}\right)\left(\frac{\delta_a}{\delta_c} - 1\right)\right) \qquad (36)$$

with $k = 0.5$ for propped construction

and $k = 0.3$ for un-propped construction

δ_a = deflection of steel beam acting alone

The expression gives acceptable results when $n_p/n_f \geq 0.4$

The increase in deflection can be disregarded where:

- either $n_p/n_f \geq 0.5$ or when force on connector does not exceed $0.7\,P_{RK}$ where P_{RK} is the characteristic resistance of the shear connector; and
- when the transverse rib depth is less than *80* mm.

The empirical nature of the above rules stipulated by BS.5950 is because of the difficulty in predicting the deflection accurately.

5.2 Effects of shrinkage of concrete and of temperature

In a dry condition, an unrestrained concrete slab is expected to shrink by *0.03%* of its length or more. But in case of composite beam the slab is restrained by steel beam. The shear connectors resist the force arising out of shrinkage, by inducing a tensile force on concrete. This reduces the apparent shrinkage of composite beam than the free shrinkage. Moreover no account of this force is taken in design as it acts in the direction opposite to that caused by load. However, the increase in deflection due to shrinkage may be significant. In an approximate approach the increase in deflection in a simply supported beam is taken as the long-term deflection due to weight of the concrete slab acting on the composite member.

Generally the span/depth ratios specified by codes take care of the shrinkage deflection. However, a check on shrinkage deflection should be done in case of thick slabs resting on small steel beams, electrically heated floors and concrete mixes with high "*free shrinkage"*. Eurocode 4 recommends that the effect of shrinkage should be considered when the span/depth ratio exceeds *20* and the free shrinkage strain exceeds *0.04%*. For dry environments, the limit on free shrinkage for normal- weight concrete is *0.0325%* and for lightweight concrete *0.05%.*

5.3 Vibration

5.3.1 General

Generally, human response to vibration is taken as the yardstick to limit the amplitude and frequency of a vibrating floor. The present discussion is mainly aimed at design of an office floor against vibration. To design a floor structure, only the source of vibration near or on the floor need be considered. Other sources such as machines, lift or cranes should be isolated from the building. In most buildings following two cases are considered-

i) People walking across a floor with a pace frequency between *1.4 Hz* and *2.5 Hz*.
ii) An impulse such as the effect of the fall of a heavy object.

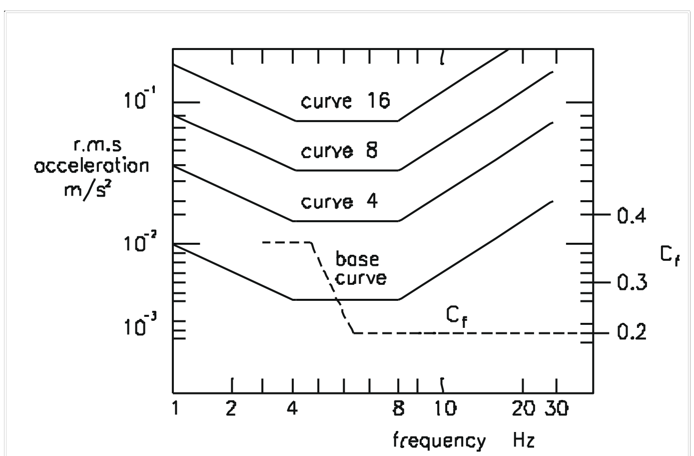


Fig.19. Curves of constant human response to vibration, and Fourier component factor

BS 6472 present models of human response to vibration in the form of a base curve as in Fig. (19). Here root mean square acceleration of the floor is plotted against its natural frequency f_0 for acceptable level *R* based on human response for different situations such as, hospitals, offices etc. The human response *R=1* corresponds to a *"minimal level of adverse comments from occupants"* of sensitive locations such as hospital, operating theatre and precision laboratories. Curves of higher response *(R)* values are also shown in the Fig. (19). The recommended values of *R* for other situations are

R = 4 for offices
R = 8 for workshops

These values correspond to continuous vibration and some relaxation is allowed in case the vibration is intermittent.

5.3.2 Natural frequency of beam and slab

The most important parameter associated with vibration is the natural frequency of floor. For free elastic vibration of a beam or one way slab of uniform section the fundamental natural frequency is,

$$f_0 = K\left(\frac{EI}{mL^4}\right)^{1/2} \qquad (37)$$

where, $K = \pi/2$ for simple support; and
$K = 3.56$ for both ends fixed.
EI = Flexural rigidity (per unit width for slabs)
L = span
m = vibrating mass per unit length (beam) or unit area (slab).

The effect of damping, being negligible has been ignored.

Un-cracked concrete section and dynamic modulus of elasticity should be used for concrete. Generally these effects are taken into account by increasing the value of *I* by *10%* for variable loading. In absence of an accurate estimate of mass (*m)*, it is taken as the mass of the characteristic permanent load plus *10%* of characteristic variable load. The value of f_0 for a single beam and slab can be evaluated in the following manner.
The mid-span deflection for simply supported member is,

$$\delta_m = \frac{5mgL^4}{384EI} \qquad (38)$$

Substituting the value of *'m'* from Eqn. (38) in Eqn. (37) we get,

$$f_0 = \frac{17.8}{\sqrt{\delta_m}} \qquad (39)$$

where, δ_m is in millimeters.

However, to take into account the continuity of slab over the beams, total deflection δ in considered to evaluate f_0, so that,

$$f_0 = \frac{17.8}{\sqrt{\delta}} \qquad (40)$$

where, $\delta = \delta_b + \delta_s$
δ_s – deflection of slab relative to beam
δ_b- deflection of beam.

From Equation. (39) and (40)

$$\frac{1}{f_0^2} = \frac{1}{f_{0s}^2} + \frac{1}{f_{0b}^2} \qquad (41)$$

where f_{os} and f_{ob} are the frequencies for slab and beam each considered alone.

From Eqn. (41) we get,

$$f_{ob} = \frac{\pi}{2}\left(\frac{EI_b}{msL^4}\right)^{1/2} \qquad (42)$$

$$f_{os} = 3.56\left(\frac{EI_S}{ms^4}\right)^{1/2} \qquad (43)$$

where, s is the spacing of the beams.

A typical vibrating profile of a floor structure is shown in Fig. (20).

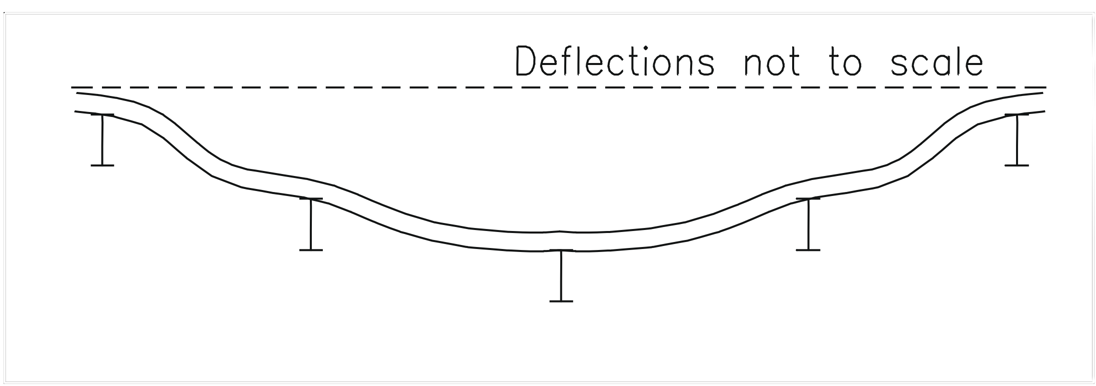


Fig.20. Cross-section of vibrating floor structure showing typical fundamental mode

5.3.3 Response factor

Reactions on floors from people walking have been analyzed by Fourier Series. It shows that the basic fundamental component has amplitude of about *240N*. To avoid resonance with the first harmonics it is assumed that the floor has natural frequency $f_0 > 3$, whereas the excitation force due to a person walking has a frequency *1.4* Hz to *2* Hz. The effective force amplitude is,

$$\overline{F} = 240 C_f \qquad (44)$$

where C_f is the Fourier component factor. It takes into account the differences between the frequency of the pedestrians' paces and the natural frequency of the floor. This is given in the form of a function of f_0 in Fig. (19).

The vertical displacement *y* for steady state vibration of the floor is given approximately by,

$$y = \frac{\overline{F}}{2 k_e \zeta} \sin 2\pi\, f_0 t \qquad (45)$$

$$\begin{aligned} \textit{where}\ \frac{\overline{F}}{k_e} &= \textit{Static deflection of floor} \\ \frac{1}{2\zeta} &= \textit{magnification factor at resonance} \\ &= 0.03\ \textit{for open plan offices with composite floor} \\ f_0 &= \textit{steady state vibration frequency of the floor} \end{aligned}$$

R.m.s value of acceleration

$$a_{r.m.s} = 4\pi^2 f_0^2 \frac{\overline{F}}{2\sqrt{2}\, k_e \zeta} \qquad (46)$$

The effective stiffness k_e depends on the vibrating area of floor, $L \times S$. The width *S* is computed in terms of the relevant flexural rigidities per unit width of floor which are I_s for slab and I_b/s for beam.

$$S = 4.5 \left(\frac{EI_s}{m f_0^{\,2}} \right)^{1/4} \qquad (47)$$

As f_{0b} is much greater than f_{0s}, the value of f_{0b} can be approximated as f_0. So, replacing $m f_0^2$ from Eqn. (42) in Eqn. (45), we get,

$$\frac{S}{L} = 3.6\left(\frac{I_s s}{I_b}\right)^{1/4} \qquad (48)$$

Eqn. (48) shows that the ratio of equivalent width to span increases with increase in ratio of the stiffness of the slab and the beam.

The fundamental frequency of a spring-mass system,

$$f_0 = \frac{1}{2\pi}\left(\frac{k_e}{M_e}\right)^{1/2} \qquad (49)$$

where, M_e is the effective mass = $mSL/4$ (approximately)

From Eqn. (51),

$$k_e = \pi^2 f_0^{\,2}\, mSL \qquad (50)$$

Substituting the value of k_e from Eqn. (50) and F from Eqn. (44) into Eqn. (46)

From definition, Response factor,

$$a_{rms} = 340\frac{C_f}{msL\zeta} \qquad (51)$$

$$a_{rms} = 5\times 10^{-3} R \quad m/s^2 \qquad (52)$$

Therefore, from Equation (52),

To check the susceptibility of the floor to vibration the value of R should be compared with the target response curve as in Fig. (19).

$$R = 68000\frac{C_f}{msL\zeta} \qquad in\ \ MKS\ \ units \qquad (53)$$

6.0 CONCLUSION

This chapter mainly deals with the theory of composite beam and the underlying philosophy behind its evolution. This comparatively new method of construction quickly gained popularity in the Western World because of its applicability in bridges, multistoried buildings, car parks etc with reduced construction time. There were valuable research studies, to support the design basis. It has been reported that saving in high yield strength steel can be up to 40% in composite construction. However this method is cost effective for larger span and taller buildings. The design procedures of simply supported as well as continuous beams have been elaborately discussed with examples in the next chapter.

7.0 REFERENCES

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