18 BEAMS SUBJECTED TO TORSION AND BENDING - II

1.0 INTRODUCTION

In the previous chapter, the basic theory governing the behaviour of beams subjected to torsion was discussed. A member subjected to torsional moments would twist about a longitudinal axis through the shear centre of the cross section. It was also pointed out that when the resultant of applied forces passed through the longitudinal shear centre axis no torsion would occur. In general, torsional moments would cause twisting and warping of the cross sections.

When the torsional rigidity *(GJ)* is very large compared with its warping rigidity $(E\Gamma)$, the section would effectively be in uniform torsion and warping moment would unlikely to be significant from the designer's perspective. Examples of this behaviour are closed hot-rolled sections (e.g. rectangular or square hollow sections) and rolled angles and Tees. Note that warping moment is developed only if warping deformation is restrained. Warping deformation in angle and *T*-sections are not small, only warping moment would be small. On the other hand, most thin walled open sections have much smaller torsional rigidity *(GJ)* compared with warping rigidity $(E\Gamma)$ values and these sections will be exhibiting significant warping moment. Hot rolled *I* sections and *H* sections would exhibit torsional behaviour in-between these two extremes and the applied loading is resisted by a combination of uniform torsion and warping torsion.

2.0 DESIGNING FOR TORSION IN PRACTICE

Any structural arrangement in which the loads are transferred to an *I* beam by torsion is <u>not</u> an efficient one for resisting loads. The message for the designers is "Avoid Torsion - if you can ". In a very large number of practical designs, the loads are usually applied in a such a manner that their resultant passes through the centroid. If the section is doubly symmetric (such as *I* or *H* sections) this automatically eliminates torsion, as the shear centre and centroid of the symmetric cross section coincide. Even otherwise load transfer through connections may - in many cases - be regarded as ensuring that the loads are effectively applied through the shear centre, thus eliminating the need for designing for torsion. Furthermore, in situations where the floor slabs are supported on top flanges of channel sections, the loads may effectively be regarded as being applied through the shear centre since the flexural stiffness of the attached slab prevents torsion of the channel.

Where significant eccentricity of loading (which would cause torsion) is unavoidable, alternative methods of resisting torsion efficiently should be investigated. These include design using box sections, tubular (hollow) sections or lattice box girders which are fully triangulated on all faces. All these are more efficient means of resisting torsional moments compared with *I* or *H* sections. Unless it is essential to utilise the torsional resistance of an *I* section, it is not necessary to take account of it. The likely torsional effects due to a particular structural arrangement chosen should be considered in the early stages of design, rather than left to the final stages, when perhaps an inappropriate member has already been chosen.

3.0 PURE TORSION AND WARPING

In the previous chapter, the concepts of uniform torsion and warping torsion were explained and the relevant equations derived.

When a torque is applied only at the ends of a member such that the ends are free to warp, then the member would develop only pure torsion.
The total angle of twist (ϕ) over a length of z is given by

$$\phi = \frac{T_q \cdot z}{G\,J} \qquad (1)$$

where T_q = applied torque
GJ = Torsional Rigidity

When a member is in non-uniform torsion, the rate of change of angle of twist will vary along the length of the member. The warping shear stress (τ_w) at a point is given by

$$\tau_w = -\frac{E\,S_{wms}\,\phi'''}{t} \qquad (2)$$

where E = Modulus of elasticity
S_{wms} = Warping statical moment at a particular point S chosen.

The warping normal stress (σ_w) due to bending moment in-plane of flanges (bi-moment) is given by

$$\sigma_w = -E\,.W_{nwfs}\,.\,\phi''$$

where W_{nwfs} = Normalised warping function at the chosen point S.

4.0 COMBINED BENDING AND TORSION

There will be some interaction between the torsional and flexural effects, when a load produces both bending and torsion. The angle of twist ϕ caused by torsion would be amplified by bending moment, inducing additional warping moments and torsional shears. The following analysis was proposed by Nethercot, Salter and Malik in reference (2).

4.1 Maximum Stress Check or "Capacity check"

The maximum stress at the most highly stressed cross section is limited to the design strength (f_y/γ_m). Assuming elastic behaviour and assuming that the loads produce bending about the major axis in addition to torsion, the longitudinal direct stresses will be due to three causes.

$$\left.\begin{aligned} \sigma_{bx} &= \frac{M_x}{Z_x} \\ \sigma_{byt} &= \frac{M_{yt}}{Z_y} \\ \sigma_w &= E.W_{nwfs}.\phi'' \end{aligned}\right\} \tag{3}$$

σ_{byt} is dependent on M_{yt}, which itself is dependent on the major axis moment M_x and the twist ϕ.

$$M_{yt} = \phi \, M_x \tag{4}$$

Thus the "capacity check" for major axis bending becomes:

$$\sigma_{bx} + \sigma_{byt} + \sigma_w \le f_y/\gamma_m. \tag{5}$$

Methods of evaluating ϕ, ϕ', ϕ'' and ϕ''' for various conditions of loading and boundary conditions are given in reference (2).

4.2 Buckling Check

Whenever lateral torsional buckling governs the design (i.e. when p_b is less than f_y) the values of σ_w and σ_{byt} will be amplified. Nethercot, Salter and Malik have suggested a simple "buckling check" along lines similar to *BS 5950, part 1*

$$\frac{\overline{M}_x}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{(f_y/\gamma_m)}\left[1 + 0.5\frac{\overline{M}_x}{M_b}\right] \le 1 \tag{6}$$

where $\overline{M_x}$, equivalent uniform moment = $m_x M_x$

and M_b, the buckling resistance moment = $\dfrac{M_E M_p}{\phi_B + \left(\phi_B^2 - M_E \, M_p\right)^{1/2}}$

in which $\phi_B = \dfrac{M_p + (\eta_{LT} + 1) M_E}{2}$

M_P , the plastic moment capacity $= f_y \,.\, Z_p / \gamma_m$

Z_p = the plastic section modulus

M_E , the elastic critical moment $= \dfrac{M_p \, \pi^2 \, E}{\lambda_{LT}^{\,2} \,.\, f_y / \gamma_m}$

where λ_{LT} is the equivalent slenderness.

4.3 Applied loading having both Major axis and Minor axis moments

When the applied loading produces both major axis and minor axis moments, the "capacity checks" and the "buckling checks" are modified as follows:

Capacity check:

$$\sigma_{bx} + \sigma_{byt} + \sigma_w + \sigma_{by} \le f_y/\gamma_m \qquad (7)$$

Buckling check:

$$\frac{\overline{M}_x}{M_b} + \frac{\overline{M}_y}{f_y Z_y / \gamma_m} + \frac{(\sigma_{byt} + \sigma_w)}{(f_y / \gamma_m)}\left[1 + 0.5\frac{\overline{M}_x}{M_b}\right] \le 1 \qquad (8)$$

where $\overline{M}_y = m_y M_y$

$\sigma_{byt} = M_y / Z_y$

4.4 Torsional Shear Stress

Torsional shear stresses and warping shear stresses should also be amplified in a similar manner:

$$\tau_{vt} = (\tau_t + \tau_w)\left(1 + 0.5\,\frac{\overline{M}_x}{M_b}\right) \qquad (9)$$

This shear stress should be added to the shear stresses due to bending in checking the adequacy of the section.

5.0 DESIGN METHOD FOR LATERAL TORSIONAL BUCKLING

The analysis for the lateral torsional buckling is very complex because of the different types of structural actions involved. Also the basic theory of elastic lateral stability cannot be directly used for the design purpose because

- the formulae for elastic critical moment M_E are too complex for routine use and
- there are limitations to their extension in the ultimate range

A simple method of computing the buckling resistance of beams is given below. In a manner analogous to the Perry-Robertson Method for columns, the buckling resistance moment, M_b, is obtained as the smaller root of the equation

$$(M_E - M_b)\,(M_p - M_b) = \eta_{LT}.\, M_E\, M_b \tag{10}$$

As explained in page 3, M_b is given by,

$$M_b = \frac{M_E\, M_p}{\phi_B + \left(\phi_B^{\,2} - M_E\, M_p\right)^{1/2}}$$

where $$\phi_B = \frac{M_p + (\eta_{LT} + 1) M_E}{2}$$

[As defined above, M_E = Elastic critcal moment

M_p = $f_y \,.\, Z_p / \gamma_m$

η_{LT} = Perry coefficient, similar to column buckling coefficient

Z_p = Plastic section modulus]

In order to simplify the analysis, *BS5950: Part 1* uses a curve based on the above concept (Fig. 1) (similar to column curves) in which the bending strength of the beam is expressed as a function of its slenderness (λ_{LT}). The design method is explained below.

The buckling resistance moment M_b is given by

$$M_b = p_b \,.Z_p \tag{11}$$

where p_b = bending strength allowing for susceptibility to lateral -torsional buckling.

Z_p = plastic section modulus.

It should be noted that $p_b = f_y$ for low values of slenderness of beams and the value of p_b drops, as the beam becomes longer and the beam slenderness, calculated as given below, increases. This behaviour is analogous to columns.

The beam slenderness (λ_{LT}) is given by,

$$\lambda_{LT} = \sqrt{\pi^2 \frac{E}{f_y}} \cdot \bar{\lambda}_{LT} \tag{12}$$

where $$\bar{\lambda}_{LT} = \sqrt{\frac{M_p}{M_E}}$$

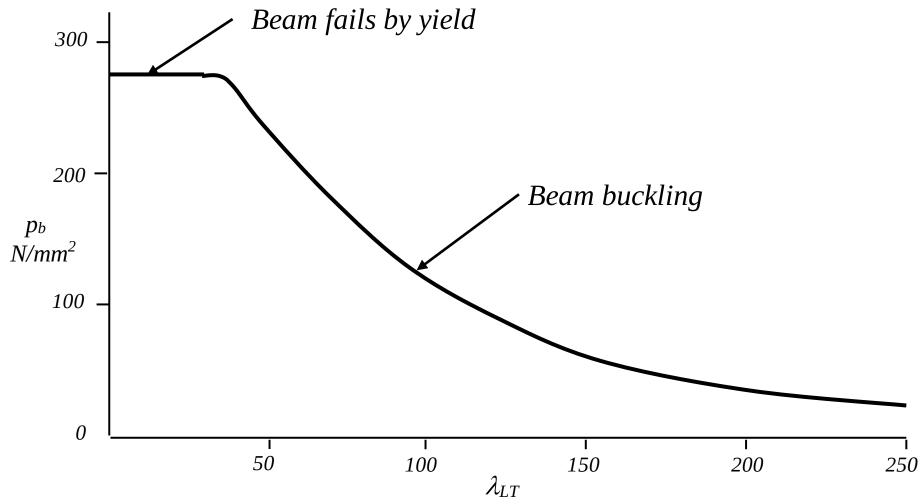


Fig.1 Bending strength for rolled sections of design strength 275 N/mm ^{2} ***according to BS 5950***

Fig. 2 is plotted in a non-dimensional form comparing the observed test data with the two theoretical values of upper bounds, viz. M_p and M_E. The test data were obtained from a typical set of lateral torsional buckling data, using hot-rolled sections. In Fig. 2 three distinct regions of behaviour can be observed:-

- stocky beams which are able to attain the plastic moment M_p, for values of $\bar{\lambda}_{LT}$ below about 0.4.
- Slender beams which fail at moments close to M_E, for values of $\bar{\lambda}_{LT}$ above about 1.2
- beams of intermediate slenderness which fail to reach either M_p or M_E . In this case $0.4 < \bar{\lambda}_{LT} < 1.2$

Beams having short spans usually fail by yielding. So lateral stability does not influence their design. Beams having long spans would fail by lateral buckling and these are termed "slender". For the practical beams which are in the intermediate range without lateral restraint, design must be based on considerations of inelastic buckling.

In the absence of instability, eqn. 11 permits that the value of f_y can be adopted for the full plastic moment capacity p_b for $\lambda_{LT} < 0.4$. This corresponds to λ_{LT} values of around 37 (for steels having $f_y = 275\ N/mm^2$) below which the lateral instability is <u>NOT</u> of concern.

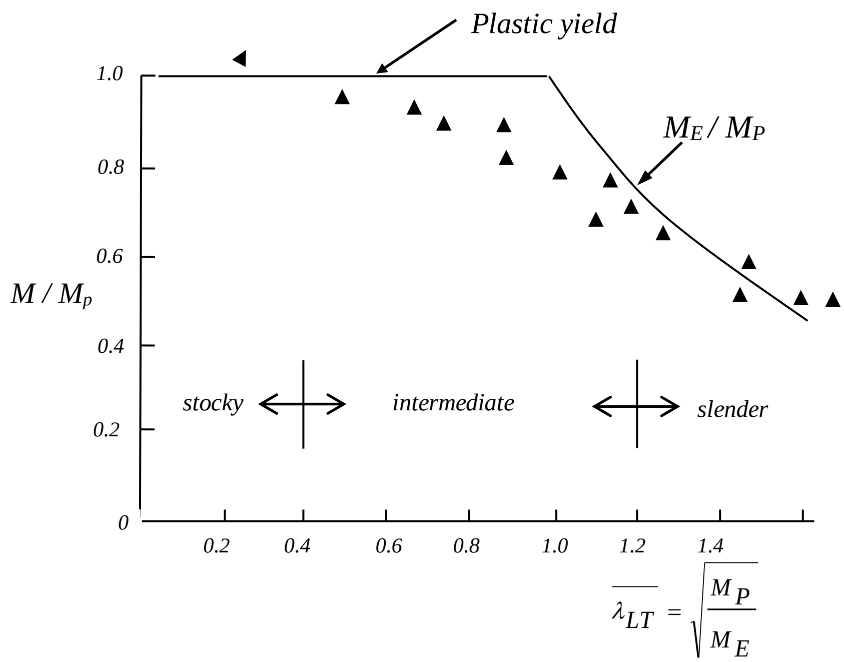


Fig.2 Comparison of test data (mostly I sections) with theoretical elastic critical moments

For more slender beams, p_b is a function of λ_{LT} which is given by ,

$$\lambda_{LT} = uv\frac{\ell}{r_y} \tag{13}$$

u is called the buckling parameter and x, the torsional index.

For flanged sections symmetrical about the minor axis,

$$u = \left(\frac{4\,Z_p{}^2\,\gamma}{A^2\,h_s{}^2}\right)^{1/4} \quad \text{and} \quad x = 0.566\ h_s\left(A/J\right)^{1/2}$$

For flanged sections symmetrical about the major axis

$$u = \left(\frac{I_y \cdot Z_p{}^2\gamma}{A^2\Gamma}\right)^{1/4} \quad \text{and} \quad x = 1.132\left(\frac{A\Gamma}{I_y\,J}\right)^{1/2}$$

In the above Z_p = plastic modulus about the major axis

$$\gamma = \left(1 - \frac{I_y}{I_x}\right)$$

A = cross sectional area of the member

Γ = torsional warping constant $\cong \dfrac{h_s^2 t_1 t_2 b_1^3 b_2^3}{12\left(t_1 b_1^3 + t_2 b_2^3\right)}$

J = the torsion constant

h_s = the distance between the shear centres of the flanges

t_1, t_2 = flange thicknesses

b_1, b_2 = flange widths

We can assume

u = *0.9* for rolled *UBs, UCs, RSJs* and channels

= *1.0* for all other sections.

$v = a\ function\ of \left(\dfrac{\ell}{r_y}, x\right)$ is given in Table 14 of *BS5950: Part I*

(for a preliminary assessment $v = 1$)

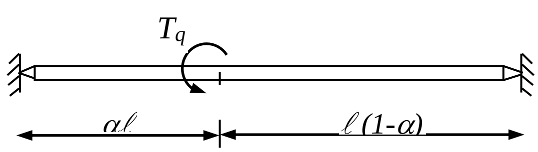
$x = D/T$ providing the above values of u are used.

5.1 Unequal flanged sections

For unequal flanged sections, eqn. 11 is used for finding the buckling moment of resistance. The value of λ_{LT} is determined by eqn.13 using the appropriate section properties. In that equation u may be taken as *1.0* and v includes an allowance for the degree of monosymmetry through the parameter $N = I_c / (I_c + I_t)$. Table 14 of *BS5950: Part* 1 must now be entered with $(\ell_E / r_y)/x$ and N.

5.2 Evaluation of differential equations

For a member subjected to concentrated torque with torsion fixed and warping free condition at the ends (torque applied at varying values of αL), the values of ϕ and its differentials are given by



For $0 \le z \le \alpha\ell$, $\quad \phi = \dfrac{T_q . a}{GJ}\left\{(1-\alpha)\dfrac{z}{a} + \left[\dfrac{sinh\dfrac{\alpha\ell}{a}}{tanh\dfrac{\ell}{a}} - cosh\dfrac{\alpha\ell}{a}\right] sinh\dfrac{z}{a}\right\}$

$$\phi' = \frac{T_q}{GJ}\left\{(1-\alpha) + \left[\frac{sinh\frac{\alpha\ell}{a}}{tanh\frac{\ell}{a}} - cosh\frac{\alpha\,\ell}{a}\right] cosh\frac{z}{a}\right\}$$

$$\phi'' = \frac{T_q}{G\,J\,a}\left[\frac{sinh\frac{\alpha\ell}{a}}{tanh\frac{\ell}{a}} - cosh\frac{\alpha\ell}{a}\right] sinh\frac{z}{a}$$

$$\phi''' = \frac{T_q}{G\,J\,a^2}\left[\frac{sinh\frac{\alpha\ell}{a}}{tanh\frac{\ell}{a}} - cosh\frac{\alpha\ell}{a}\right] cosh\frac{z}{a}$$

Similar equations are available for different loading cases and for different values of $\alpha\ell$. Readers may wish to refer Ref. (2) for more details. We are unable to reproduce these on account of copyright restrictions.

6.0 SUMMARY

This chapter is aimed at explaining a simple method of evaluating torsional effects and to verify the adequacy of a chosen cross section when subjected to torsional moments. The method recommended is consistent with BS 5950: Part 1.

7.0 REFERENCES

(1) British Standards Institution, BS 5950: Part 1: 1985. Structural use of steelwork in Building part 1: Code of Practice for design in simple and continuous construction: hot rolled sections. BSI, 1985.

(2) Nethercot, D. A., Salter, P. R., and Malik, A. S. Design of Members Subject to Combined Bending and Torsion , The Steel construction Institute , 1989.

(3) Steelwork design guide to BS 5950: Part 1 1985, Volume 1 Section properties and member capacities. The Steel Construction Institute, 1985.

(4) Introduction to Steelwork Design to BS 5950: Part 1, The Steel Construction Institute, 1988.

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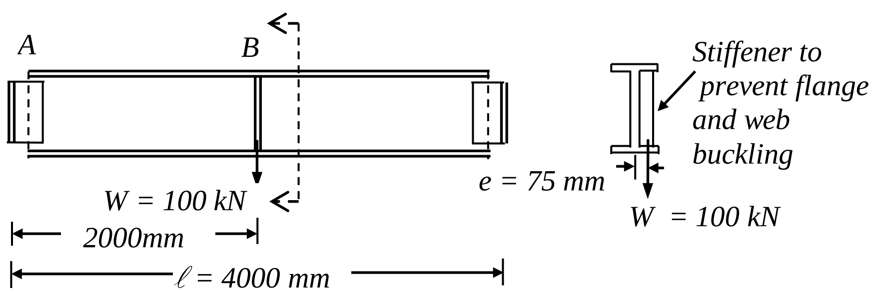
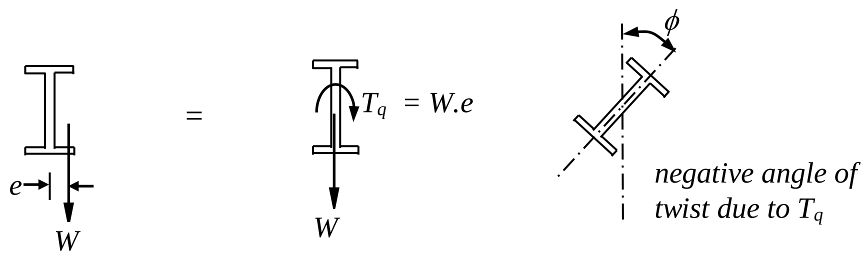
<u>Example 1</u>

The beam shown below is unrestrained along its length. An eccentric load is applied to the bottom flange at the centre of the span in such a way that it does not provide any lateral restraint to the member.
The end conditions are assumed to be simply supported for bending and fixed against torsion but free for warping. For the factored loads shown, check the adequacy of the trial section.



Replace the actual loading by an equivalent arrangement, comprising a vertical load applied through the shear centre and a torsional moment as shown below.



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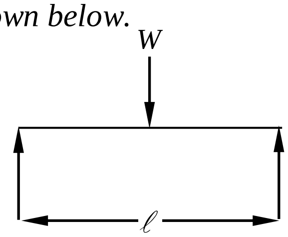
Loadings due to plane bending and torsion are shown below.



(i) Plane

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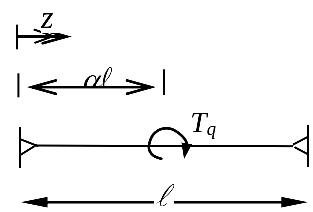

(ii) Torsional

<u>Loading (Note: These are factored loads and are not to be multiplied by γ_f)</u>

Point load, $W = 100$ *kN*
Distributed load (self weight), $w = 1$ *kN/m (say)*
Eccentricity, $e = 75$ *mm*

<u>Bending effects (at U.L.S)</u>

Moment at B, $M_{xB} = 102$ *kNm*
Shear at A, $F_{vA} = 52$ *kN*
Shear at B, $F_{vB} = 50$ *kN*

<u>Torsional effects (at U.L.S)</u>

Torsional moment, $T_q = W.e$
$T_q = 100 \times 75 \times 10^{-3} = 7.5$ *kNm*
This acts in a negative sense, $\therefore T_q = -7.5$ *kNm*

Generally wide flange sections are preferable to deal with significant torsion. In this example, however, an ISWB section will be tried.

<u>Try ISWB 500 × 250 @ 95.2 kg/m</u>
<u>Section properties from steel tables.</u>

Depth of section $D = 500$ *mm*
Width of section $B = 250$ *mm*

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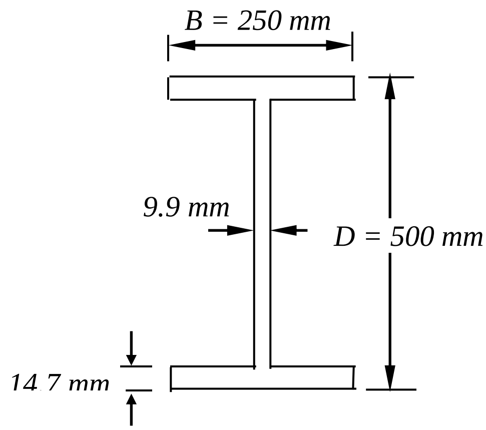

Web thickness $t = 9.9\ mm$
Flange thickness $T = 14.7\ mm$
Moment of inertia $I_{xx} = 52291\ cm^4$
Moment of inertia $I_{yy} = 2988\ cm^4$
Radius of gyration $r_y = 49.6\ mm$
Elastic modulus $Z_x = 2092\ cm^3$
Elastic modulus $Z_y = 239\ cm^3$
Cross sectional area $A = 121.2\ cm^2$

<u>Additional properties</u>

Torsional constant, $J = \frac{1}{3}\left[2BT^3 + (D - 2T)\,t^3\right]$

$$= \frac{1}{3}\left[2\times 250\times 14.7^3 + (500 - 2\times 14.7)\,9.9^3\right] = 682\times 10^3\ mm^4$$

Warping constant, $\Gamma = \frac{I_y \cdot h^2}{4}$

$$= \frac{2988\times 10^4 \times (500 - 14.7)^2}{4} = 1.76\times 10^{12}\ mm^6$$

Shear modulus, $G = \frac{E}{2(1+\upsilon)}$

$$= \frac{2\times 10^5}{2(1+0.3)} = 76.9\ kN/mm^2$$

Torsional bending constant, $a = \left(\frac{E\Gamma}{GJ}\right)^{1/2}$

$$= \left(\frac{2\times 10^5 \times 1.76\times 10^{12}}{76.9\times 10^3 \times 682\times 10^3}\right)^{1/2} = 2591\ mm$$

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Normalized warping function,

$$W_{nwfs} = \frac{h\,B}{4}$$

$$= \frac{(500 - 14.7)\times 250}{4}$$

$$= 30331 \ \ mm^2$$

Warping statical moment,

$$S_{wms} = \frac{h\,B^2\,T}{16}$$

$$= \frac{485.3 \times 250^2 \times 14.7}{16}$$

$$= 2787 \times 10^4 \ mm^4$$

Statical moment for flange,

$$Q_f = A_f \,.\, y_f$$

$$= (120.05 \times 14.7) \times 242.7$$

$$= 428.2 \times 10^3 \ mm^3$$

Statical moment for web,

$$Q_w = (A/2) \times y_w$$

$$y_w = \frac{\left[14.7 \times 250 \times 242.7 + 9.9 \times 235.3 \times \dfrac{235.3}{2}\right]}{14.7 \times 250 + 9.9 \times 235.3} = 194.2 \ mm$$

$$\therefore \quad Q_w = 6061 \times 194.2$$

$$= 1166 \times 10^3 \ mm^3$$

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Material Properties $\frac{250}{1.15} \times \left[\frac{250 \times 500^2}{4} - \frac{240.1 \times 470.6^2}{4} \right] = 507 kNm$

Shear modulus, $G = 76.9\ kN/mm^2$

Design strength, $p_y = 250 / \gamma_m = 250 / 1.15 = 217\ N/mm^2$

<u>Check for Combined bending and torsion</u>

(i) *<u>Buckling check (at Ultimate Limit State)</u>*

$$\frac{\overline{M_x}}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{f_y / \gamma_m} \left[1 + 0.5 \frac{\overline{M_x}}{M_b} \right] \le 1$$

$$\overline{M_x} = m \times M_{xB}$$

$$m = 1.0$$

$$\therefore \quad \overline{M_x} = 1.0 \times M_{xB} = 102\ kNm$$

Effective length $\ell_E = 1.0\,L$ $\quad \ell_E = 4000\ mm$

The buckling resistance moment, $$M_b = \frac{M_E M_p}{\phi_B + \left(\phi_B^2 - M_E M_p\right)^{1/2}}$$

$$\phi_B = \frac{M_p + (\eta_{LT} + 1) M_E}{2}$$

BS 5950: Part I App.B.2

where

M_E = *elastic critical moment*

M_p = *plastic moment capacity*

$= f_y.Z_p / \gamma_m =$

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Elastic critical moment, $M_E = \dfrac{M_p \, \pi^2 \, E}{\lambda_{LT}^{\,2} \cdot p_y}$

BS 5950: Part I App.B.2.2

λ_{LT} = *the equivalent slenderness* = $nuv\,\lambda$

λ = *the minor axis slenderness* = $\ell_E / r_y = 4000 / 49.6 = 80.7$

$n = 0.86, \quad u = 0.9$

BS 5950: Part I Table 14

v = *slenderness factor (according to N and* λ/x*)*

$$N = \frac{I_{cf}}{I_{cf} + I_{tf}} = 0.5 \text{ (for equal flanged sections)}$$

BS 5950: Part 1 App.B.2.5

$$x = 1.132 \left(\frac{A\Gamma}{I_y \, . J} \right)^{1/2}$$

$$= 1.132 \left(\frac{12122 \times 1.76 \times 10^{12}}{2988 \times 10^4 \times 681.6 \times 10^3} \right)^{1/2} = 36.63$$

$$\lambda / x = 80.7 / 36.6 = 2.2$$

$$v = 0.948$$

$$\lambda_{LT} = nuv\lambda$$

$$= 0.86 \times 0.9 \times 0.948 \times 80.7 = 59.2$$

$$M_E = \frac{583 \times 10^6 \times \pi^2 \times 2 \times 10^5}{59.2^2 \times 217}$$

$$= 1143 \text{ kNm}$$

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BS 5950: Part 1 App.B.2.3

$$\phi_B = \frac{M_p + (\eta_{LT} + 1)M_E}{2}$$

The Perry coefficient, $\eta_{LT} = \alpha_b(\lambda_{LT} - \lambda_{LO})$

Limiting equivalent slenderness, $\lambda_{LO} = 0.4\left(\frac{\pi^2 E}{p_y}\right)^{1/2}$

$$= 0.4\left(\frac{\pi^2 \times 2 \times 10^5}{217}\right)^{1/2} = 38.2$$

$$\eta_{LT} = 0.007\,(59.2 - 38.2) = 0.15$$

$$\therefore \quad \phi_B = \frac{507 + (0.15 + 1) \times 1143}{2} = 911 \text{ kNm}$$

$$\therefore \quad M_b = \frac{M_E M_p}{\phi_B + \left(\phi_B^2 - M_E M_p\right)^{1/2}}$$

$$= \frac{1143 \times 507}{911 + \left(911^2 - 1143 \times 507\right)^{1/2}} = 411 \text{ kNm}$$

$$M_{yt} = M_x \cdot \phi$$

<u>*To calculate ϕ*</u>

$$\ell / a = 4000 / 2591 = 1.54$$

$$z = \alpha \ell, \quad \alpha = 0.5$$

$$= 0.5 \times 4000 = 2000$$

$$\alpha \ell / a = 0.77$$

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$$\phi = \frac{T_q\,.a}{GJ}\left\{(1-\alpha)\frac{z}{a} + \left[\frac{\sinh\frac{\alpha\ell}{a}}{\tanh\frac{\ell}{a}} - \cosh\frac{\alpha\ell}{a}\right]\sinh\frac{z}{a}\right\}$$

Ref. 2.0 App. B

$$= \frac{-7.5\times10^6\times2591}{76.9\times10^3\times681.6\times10^3}\left\{(1-0.5)0.77 + \left[\frac{\sinh0.77}{\tanh1.54} - \cosh0.77\right]\sinh0.77\right\}$$

$$= -0.023\ rads$$

$$M_{yt} = 102\times0.023 = 2.36\ kNm$$

$$\sigma_{byt} = \frac{M_{yt}}{Z_y} = \frac{2.36\times10^6}{239\times10^3} = 9.89\ N/mm^2$$

$$\sigma_w = E\,.\,W_{nwfs}\,.\phi''$$

<u>*To calculate* ϕ''</u>

$$\phi'' = \frac{T_q}{G\,J\,a}\left[\frac{\sinh\frac{\alpha\ell}{a}}{\tanh\frac{\ell}{a}} - \cosh\frac{\alpha\ell}{a}\right]\sinh\frac{z}{a}$$

Ref. 2.0 App. B

$$= \frac{-7.5\times10^6}{76.9\times10^3\times681.6\times10^3\times2591}\left[\frac{\sinh0.77}{\tanh1.54} - \cosh0.77\right]\sinh0.77$$

$$= 1.8\times10^{-8}$$

$$\sigma_w = 2\times10^5\times30331\times1.8\times10^{-8} = 109\ N/mm^2$$

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$$\frac{\overline{M_x}}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{f_y / \gamma_m}\left[1 + 0.5\frac{\overline{M_x}}{M_b}\right] \le 1$$

$$\frac{102 \times 10^6}{411 \times 10^6} + \frac{(9.9 + 109.2)}{(250 / 1.15)}\left[1 + 0.5 \times \frac{102 \times 10^6}{411 \times 10^6}\right] = 0.86 < 1$$

$\therefore$ *Buckling is O. K*

(i) *Local "capacity" check*

$$\sigma_{bx} + \sigma_{byt} + \sigma_w \le f_y / \gamma_m$$

$$\sigma_{bx} = M_x / Z_x = 102 \times 10^6 / 2092 \times 10^3 = 48.8\ N/mm^2$$

$$\therefore\ 48.8 + 9.9 + 109.2 = 168\ N/mm^2 < 217\ N/mm^2$$

$\therefore$ *O. K*

Strictly the shear stresses due to combined bending and torsion should be checked, although these will seldom be critical.

Shear stresses due to bending (at Ultimate Limit state)

At support:-

In web, $$\tau_{bw} = \frac{F_{VA} \cdot Q_w}{I_x . t} = \frac{52 \times 10^3 \times 1166 \times 10^3}{52291 \times 10^4 \times 9.9}$$

$$= 11.7\ N/mm^2$$

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In flange, $\tau_{bf} = \dfrac{F_{VA} \cdot Q_f}{I_x . T} = \dfrac{52 \times 10^3 \times 428.2 \times 10^3}{52291 \times 10^4 \times 14.7}$

$= 2.9 \ N/mm^2$

At midspan :-

In web, $\tau_{bw} = 11.3 \ N/mm^2$

In flange, $\tau_{bf} = 2.8 \ N/mm^2$

<u>Shear stresses due to torsion (at Ultimate Limit state)</u>

Stress due to pure torsion, $\tau_t = G.t.\phi'$

Stress due to warping, $\tau_w = \dfrac{-E.S_{wms}.\phi'''}{t}$

<u>To calculate ϕ' and ϕ'''</u>

$$\phi' = \frac{T_q}{G\,J}\left\{(1-\alpha) + \left[\frac{\sinh\frac{\alpha\,\ell}{a}}{\tanh\frac{\ell}{a}} - \cosh\frac{\alpha\,\ell}{a}\right]\cosh\frac{z}{a}\right\}$$

Ref. 2.0 App.B

$$\phi''' = \frac{T_q}{G\,J\,a^2}\left[\frac{\sinh\frac{\alpha\,\ell}{a}}{\tanh\frac{\ell}{a}} - \cosh\frac{\alpha\,\ell}{a}\right]\cosh\frac{z}{a}$$

At $\alpha = 0.5$,

$\dfrac{\alpha\,\ell}{a} = \dfrac{0.5 \times 4000}{2591} = 0.77$

$\sinh\dfrac{\alpha\,\ell}{a} = 0.851, \quad \cosh\dfrac{\alpha\,\ell}{a} = 1.313, \quad \tanh\dfrac{\ell}{a} = 0.913$

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At support, $z = 0$

$$\cosh\frac{z}{a} = \cosh(0) = 1.0$$

At midspan, $z = 2000$

$$\cosh\frac{z}{a} = \cosh(0.77) = 1.313$$

<u>At support</u>

$$\phi''' = \frac{-7.5\times10^6}{76.9\times10^3\times681.6\times10^3\times2591^2}\left[\frac{0.851}{0.913} - 1.313\right]\times 1$$

$$\therefore\ \phi''' = 0.812\times10^{-11}$$

$$\phi' = \frac{-7.5\times10^6}{76.9\times10^3\times681.6\times10^3}\left[(1-0.5)+\left[\frac{0.851}{0.913} - 1.313\right]\times 1\right]$$

$$= -1.7\times10^{-5}$$

Stresses due to pure torsion.

In web, $\tau_{tw} = G.t.\phi'$

$$\tau_{tw} = 76.9\times10^3\times9.9\times(-1.7\times10^{-5})$$

$$= -12.95\ N/mm^2$$

In flange, $\tau_{tf} = G.T.\phi'$

$$\tau_{tf} = 76.9\times10^3\times14.7\times(-1.7\times10^{-5})$$

$$= -19.22\ N/mm^2$$

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Stresses due to warping in flange,

$$\tau_{wf} = \frac{-E.S_{wms}.\phi'''}{T}$$

$$\tau_{wf} = \frac{-2\times10^5\times2787\times10^4\ \times0.812\times10^{-11}}{14.7} = -3.1\,N/mm^2$$

<u>At midspan</u>

$$\phi' = 0$$

$$\phi''' = \frac{-7.5\times10^6}{76.9\times10^3\times681.6\times10^3\times2591^2}\left[\frac{0.851}{0.913}-1.313\right]\times1.313$$

$$= 1.06\ \times10^{-11}$$

Stresses due to pure torsion,

In web, $\tau_{tw} = G.t.\phi' = 0$

In flange, $\tau_{tf} = G.T.\phi' = 0$

Stresses due to warping in flange,

$$\tau_{wf} = \frac{-E.S_{wms}.\phi'''}{T}$$

$$\tau_{wf} = \frac{-2\times10^5\times2787\times10^4\ \times1.06\times10^{-11}}{14.7} = -4.02\,N/mm^2$$

By inspection the maximum combined shear stresses occur at the support.

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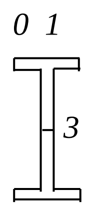

At support

$$\tau_{vt} = (\tau_t + \tau_w)\left(1 + 0.5\ \frac{\overline{M}_x}{M_b}\right)$$

In web at 3, $\tau_{tw} = -12.95\ N/mm^2$

$$\therefore \qquad \tau_{vt} = -12.95\left(1 + 0.5 \times \frac{102}{411}\right) = -14.6\ N/mm^2$$

This must be added to the shear stresses due to plane bending.

$$\tau = \tau_{bw} + \tau_{vt}$$

$$\tau = \pm\ 11.7 - 14.6 = -26.3\ N/mm^2 (\text{ acting downwards})$$

In the top flange at 1, $\tau_{tf} = -19.2\ N/mm^2$

$$\tau_{wf} = -3.1\ N/mm^2$$

$$\therefore \qquad \tau_{vt} = (-19.2 - 3.1)\left(1 + 0.5\ \frac{102}{411}\right) = -25.1\ N/mm^2$$

$$\tau = \tau_{bf} + \tau_{vt} = -27.9\ N/mm^2 (\text{ acting left to right})$$

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Shear strength, $f_v = 0.6\, f_y / \gamma_m = 0.6 \times 250 / 1.15 = 130\ N/mm^2$

Since $\tau < f_v \quad 27.9 < 130\ N/mm^2$

Section is adequate for shear

Referring back to the determination of the maximum angle of twist ϕ, in order to obtain the value at working load it is sufficient to replace the value of torque T_q with the working load value as ϕ is linearly dependent on T_q. Since T_q is due to solely the imposed point load W, dividing by the appropriate value of γ_f will give :-

$\therefore$ *Working load value of* T_q *is* $\dfrac{7.5}{1.6} = 4.7\ kNm$

the corresponding value of $\phi = \dfrac{0.026}{1.6} = 0.016\ rads = 0.93^\circ$

On the assumption that a maximum twist of 2° is acceptable at working load, in this instance the beam is satisfactory.

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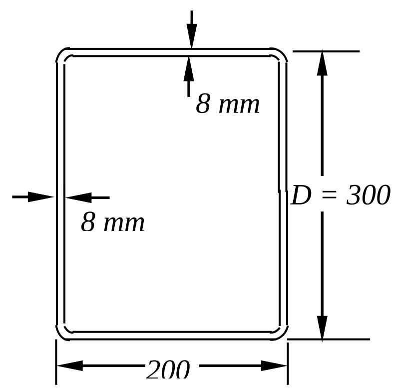
Example 2

Redesign the member shown in example 1, using a rectangular hollow section.

Try $300 \times 200 \times 8$ @ 60.5 kg / m R. H. S
Section properties.

Depth of section $D = 300$ mm
Width of section $B = 200$ mm
Web thickness $t = 8$ mm
Flange thickness $T = 8$ mm
Area of section $A = 77.1\ cm^2$
Moment of inertia $I_x = 9798\ cm^4$
Radius of gyration $r_y = 8.23$ cm
Elastic modulus $Z_x = 653\ cm^3$
Elastic modulus $Z_y = 522\ cm^3$
Plastic modulus $Z_p = 785\ cm^3$



Additional properties

Torsional constant $J = \dfrac{t^3 h}{3} + 2K \cdot A_h$

Area enclosed by the mean perimeter of the section, $A_h = (B - t)(D - T)$
(neglecting the corner radii)

$= (200 - 8)(300 - 8)$

$= 56064\ mm^2$

The mean perimeter, $h = 2[(B - t) + (D - T)]$

$= 2[(200 - 8) + (300 - 8)] = 968$ mm

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$$K = \frac{2A_h \cdot t}{h} = \frac{2 \times 56064 \times 8}{968} = 927\ mm^2$$

$$\therefore \quad \text{Torsional constant,} \quad J = \frac{8^3 \times 968}{3} + 2 \times 927 \times 56064$$

$$= 104 \times 10^6\ mm^4$$

$$\text{Torsional modulus constant,} \quad C = \frac{J}{t + K/t}$$

$$= \frac{104 \times 10^6}{8 + 927/8} = 840 \times 10^3\ mm^3$$

Material properties

$$\text{Shear modulus,} \quad G = \frac{E}{2(1+\upsilon)} = \frac{2 \times 10^5}{2(1+0.3)} = 76.9\ kN/mm^2$$

$$\text{Design strength,}\ p_y = 250/\gamma_m = 250/1.15 = 217\ N/mm^2$$

<u>Check for combined bending and torsion</u>

(i) *<u>Buckling check</u>*

$$\frac{\overline{M}_x}{M_b} + \frac{(\sigma_{byt} + \sigma_w)}{f_y/\gamma_m}\left[1 + 0.5\frac{\overline{M}_x}{M_b}\right] \le 1$$

Since slenderness ratio ($\ell_E / r_y = 4000/82.3 = 48.6$) is less than the limiting value $\left(350 \times \frac{275}{250} \times \frac{250}{f_y} = 385\right)$ given in BS 5950 Part 1, table 38, lateral torsional buckling need not be considered..

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Hence $M_b = M_{cx}$

Shear capacity $P_v = 0.6\, f_y / \gamma_m \,.\, A_v$

Shear area $A_v = \left(\dfrac{D}{D+B}\right) A = \left(\dfrac{300}{300+200}\right) 77.1 = 46.3 \text{ cm}^2$

$\therefore\ P_v = 0.6 \times (250/1.15) \times 46.3 \times 10^2 \times 10^{-3} = 604.3 \text{ kN}$

Since $F_{VB} < 0.6\, P_v$ $\quad 50 < 363$

$M_{cx} = f_y .\, Z_p / \gamma_m \le 1.2\, f_y / \gamma_m .\, Z_x$ *(for plastic sections)*

$\therefore\ M_{cx} = 1.2 \times (250/1.15) \times 653 \times 10^{-3} = 170 \text{ kNm}$ — *BS 5950: Part 1 4.2.5*

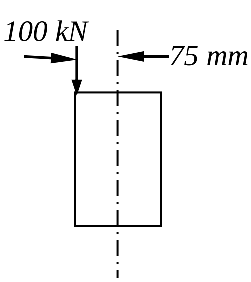
$\overline{M} = m \cdot M_{xB}$

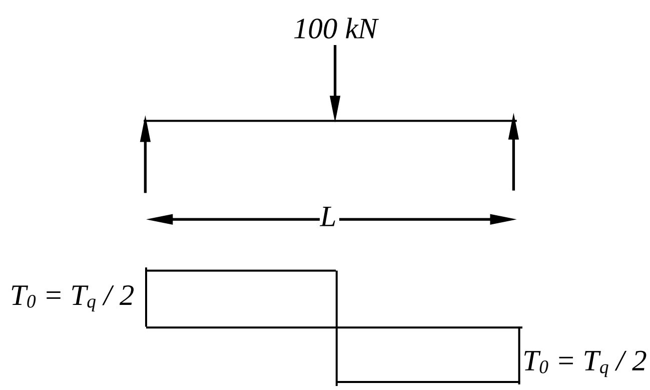
$m = 1.0$

$\overline{M} = 1.0 \times 102 = 102 \text{ kNm}$ — *BS 5950: Part 1 4.3.7.2 table 13*

<u>To calculate ϕ</u>

The 100 kN eccentric load gives a value of $T_q = 100 \times 0.75 = 7.5 \text{ kNm}$





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$$\phi = \frac{T_0}{G\,J} \cdot z$$

$$T_0 = \frac{T_q}{2} = \frac{7.5}{2} = 3.75\ kNm$$

At centre of span, $z = \ell / 2 = 2000\ mm$

$$\phi = \frac{3.75 \times 10^6 \times 2000}{76.9 \times 10^3 \times 104 \times 10^6} = 0.001\ radians$$

$$M_{yt} = \phi \,.\, M_{xB} = 0.001 \times 102 = 0.102\ kNm$$

$$\sigma_{byt} = \frac{M_{yt}}{Z_y} = \frac{0.102 \times 10^6}{522 \times 10^3} = 0.195\ N/mm^2$$

Warping stresses (σ_w *) are insignificant due to the type of section employed.*

Check becomes

$$\frac{\overline{M}_x}{M_b} + \frac{\sigma_{byt}}{f_y / \gamma_m}\left[1 + 0.5\,\frac{\overline{M}_x}{M_b}\right] \le 1$$

$$\frac{102}{170} + \frac{0.195}{250/1.15}\left[1 + 0.5 \times \frac{102}{170}\right] = 0.6 < 1$$

<u>$\therefore$ ***O. K***</u>

(ii) <u>*Local capacity check*</u>

$$\sigma_{bx} + \sigma_{byt} + \sigma_w \le f_y/\gamma_m$$

$$\sigma_{bx} = M_{xB} / Z_y$$

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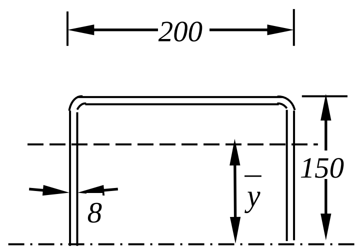
$$\sigma_{bx} = \frac{102 \times 10^6}{522 \times 10^3} = 196 \ N/mm^2$$

$$196 + 0.195 + 0 = 196.2 < 217 \ N/mm^2$$

$\therefore$ ***<u>O. K</u>***

<u>Shear stresses due to bending (at Ultimate Limit state)</u>

Maximum value occurs in the web at the support.



$$\tau_{bw} = \frac{F_{VA} . Q_w}{I_x . t_1}$$

$$Q_w = A_1 \cdot \bar{y}$$

$$A_1 = \frac{A}{2}$$

$$\bar{y} = \frac{150 \times 8 \times \frac{150}{2} \times 2 + 184 \times 8 \times 146}{A_1} \times 10^{-3} = \frac{395}{A_1} \ cm$$

$$\therefore \quad Q_w = A_1 \times \frac{395}{A_1} = 395 \ cm^3$$

$$\tau_{bw} = \frac{52 \times 10^3 \times 395 \times 10^3}{9798 \times 10^4 \times 2 \times 8} = 13.1 \ N/mm^2$$

<u>Shear stresses due to torsion (at Ultimate limit State)</u>

$$\tau_t = \frac{T_0}{C} = \frac{T_q}{2C} = \frac{7.5 \times 10^6}{2 \times 837 \times 10^3} = 4.5 \ N/mm^2$$

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Total shear stress (at Ultimate Limit State)

$$\tau = \tau_{bw} + \tau_{vt}$$

$$\tau_{vt} = (\tau_t + \tau_w)\left(1 + 0.5\,\frac{\overline{M}_x}{M_b}\right)$$

$$= (4.5 + 0)\left(1 + 0.5 \times \frac{102}{170}\right) = 5.9\ N/mm^2$$

$$\tau = 13.1 + 5.9 = 19\ N/mm^2$$

Shear strength $p_v = 0.6 f_y / \gamma_m = 0.6 \times 250/1.15 = 130\ N/mm^2$ — *BS 5950: Part 1 4. 2. 3*

Since $\tau < p_v$ $19 < 130\ N/mm^2$

$\therefore$ ***the section is adequate for shear.***