12 UNRESTRAINED BEAM DESIGN – II

1.0 INTRODUCTION

The basic theory of beam buckling was explained in the previous chapter. Doubly symmetric I- section has been used throughout for the development of the theory and later discussion. It was established that practical beams fail by:

(i) Yielding, if they are short
(ii) Elastic buckling, if they are long, or
(iii) Inelastic lateral buckling, if they are of intermediate length.

A conservative method of designing beams was also explained and its limitations were outlined.

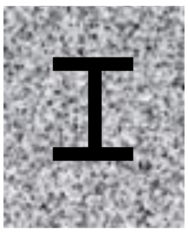
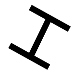
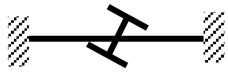
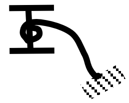
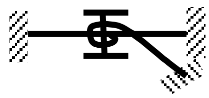
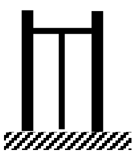
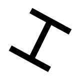
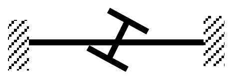
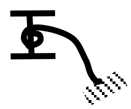
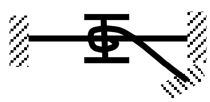
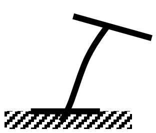
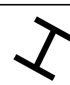
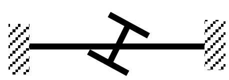
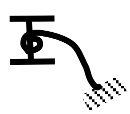
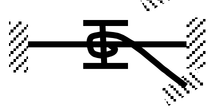
In this chapter a few cases of lateral buckling strength evaluation of beams encountered in practice would be explained. Cantilever beams, continuous beams, beams with continuous and discrete lateral restraints are considered. Cases of monosymmetric beams and non-uniform beams are covered. The buckling strength evaluation of non-symmetric sections is also described.

2.0 CANTILEVER BEAMS

A cantilever beam is completely fixed at one end and free at the other. In the case of cantilevers, the support conditions in the transverse plane affect the moment pattern. For design purposes, it is convenient to use the concept of *notional effective length, k*, which would include both loading and support effects. The notional effective length is defined as the length of the notionally simply supported (in the lateral plane) beam of similar section, which would have an elastic critical moment under uniform moment equal to the elastic critical moment of the actual beam under the actual loading conditions. Recommended values of *'k'* for a number of cases are given in Table 1. It can be seen from the values of '*k*' that it is more effective to prevent twist at the cantilever edge rather than the lateral deflection.

Generally, in framed structures, continuous beams are provided with overhang at their ends. These overhangs have the characteristics of cantilever beams. In such cases, the type of restraint provided at the outermost vertical support is most significant. Effective prevention of twist at this location is of particular importance. Failure to achieve this would result in large reduction of lateral stability as reflected in large values of '*k'*, in Table 1.

Table 1 Recommended values of 'k'

Restraint conditions		Loading condition	
At support	At tip	Normal	Destabilizing
Built in laterally and torsionally 	Free 	0.8ℓ	1.4ℓ
	Lateral restraint only 	0.7ℓ	1.4ℓ
	Torsional Restraint only 	0.6ℓ	0.6ℓ
	Lateral and Torsional Restraint 	0.5ℓ	0.5ℓ
Continuous with lateral and torsional restraint 	Free 	1.0ℓ	2.5ℓ
	Laterally restraint only 	0.9ℓ	2.5ℓ
	Torsional Restraint only 	0.8ℓ	1.5ℓ
	Laterally and Torsional Restraint 	0.7ℓ	1.2ℓ
Continuous, with lateral restraint only 	Free 	3.0ℓ	7.5ℓ
	Lateral restraint only 	2.7ℓ	7.5ℓ
	Torsional Restraint only 	2.4ℓ	4.5ℓ
	Laterally and Torsional Restraint 	2.1ℓ	3.6ℓ

For continuous cantilevers ℓ not less than ℓ_1 where ℓ_1 is the length of the adjacent span

3.0 CONTINUOUS BEAMS

Beams, extending over a number of spans, are normally continuous in vertical, lateral or in both planes. In the cases, where such continuity is not provided lateral deflection and twisting may occur. Such a situation is typically experienced in roof purlins before sheeting is provided on top of them and in beams of temporary nature. For these cases, it is always safe to make no assumption about possible restraints and to design them for maximum effective length.

Another case of interest with regard to lateral buckling is a beam that is continuous in the lateral plane i.e. the beam is divided into several segments in the lateral plane by means of fully effective braces. The buckled shape for such continuous beams include deformation of all the segments irrespective of their loading. Effective length of the segments will be equal to the spacing of the braces if the spacing and moment patterns are similar. Otherwise, the effective length of each segment will have to be determined separately.

To illustrate the behaviour of continuous beams, a single-span beam provided with equally loaded cross beams is considered (see Fig. 1).

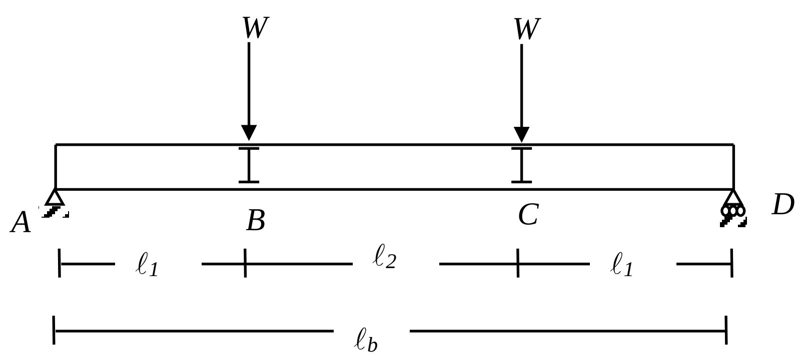


Fig. 1 Single - span beam

Two equally spaced, equally loaded cross beams divide the beam into three segments laterally. In this case, true M_{cr} of the beam and its buckling mode would depend upon the spacing of the cross beams. The critical moment M_{crB} for any ratio of ℓ_1 / ℓ_b would lie in between the critical moment values of the individual segments. The critical moments for the two segments are obtained using the basic equation given in the earlier chapter.

$$M_{cr1} = 1.75 \frac{\pi}{\ell_1} \sqrt{(E I_y GJ)} \sqrt{1 + \frac{\pi^2 E \Gamma}{\ell_1^2 \, GJ}} \qquad (1)$$

(In the outer segment, m = 0.57. Using 1 / m and the basic moment, the critical moment is determined)

$$M_{cr2} = \frac{\pi}{\ell_2} \sqrt{(EI_y GJ)} \sqrt{1 + \frac{\pi^2 E \Gamma}{\ell_2^2 \, GJ}} \qquad (2)$$

(This segment is loaded by uniform moments at its ends – basic case)

M_{cr1} and M_{cr2} values are plotted against ℓ_1 / ℓ_b and shown in Fig. 2 for the particular case considered with equal loading and a constant cross section throughout.

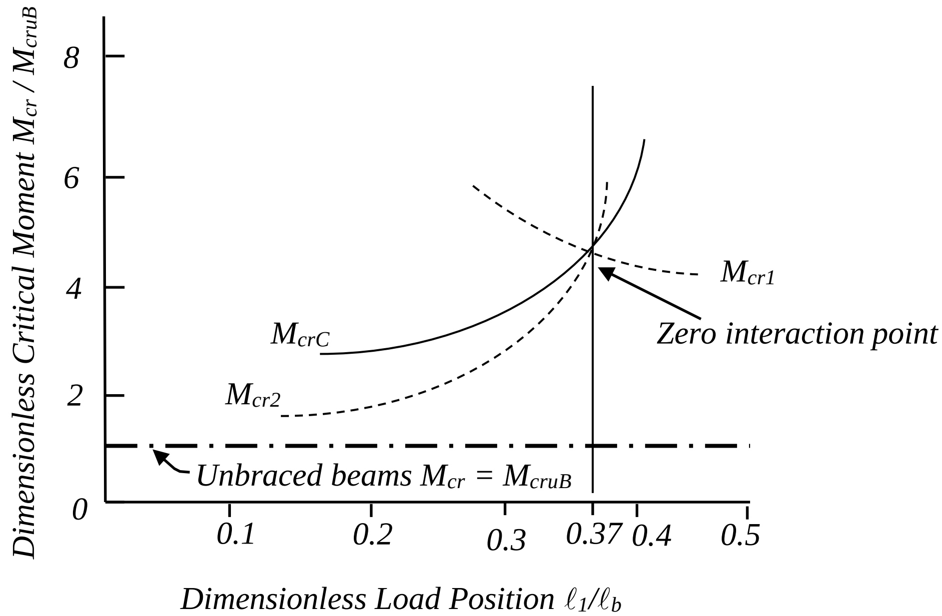


Fig.2 Interaction between M_{cr1} and M_{cr2}

It is seen that for $\ell_1 / \ell_b = 0.37$, M_{cr1} *and* M_{cr2} are equal and the two segments are simultaneously critical. The beam will buckle with no interaction between the two segments. For any other value of ℓ_1 / ℓ_b there will be interaction between the segments and the critical load would be greater than the individual values, as shown in the figure. For values of $\ell_1 / \ell_b < 0.37$, outer segments will restrain the central segment and vice-versa when $\ell_1 / \ell_b > 0.37$.

The safe load for a laterally continuous beam may be obtained by calculating all segmental critical loads individually and choosing the lowest value assuming each segment as simply supported at its ends.

It is of interest to know the behaviour of beams, which are continuous in both transverse and lateral planes. Though the behaviour is similar to the laterally unrestrained beams, their moment patterns would be more complicated. The beam would buckle in the lateral plane and deflect in the vertical plane. There is a distinct difference between the points of contraflexure in the *buckled shape* and points of contraflexure in the *deflected shape*. These points will not normally occur at the same location within a span, as shown in Fig. 3. Therefore, it is wrong to use the distance between the points of contraflexure of the deflected shape as the effective length for checking buckling strength.

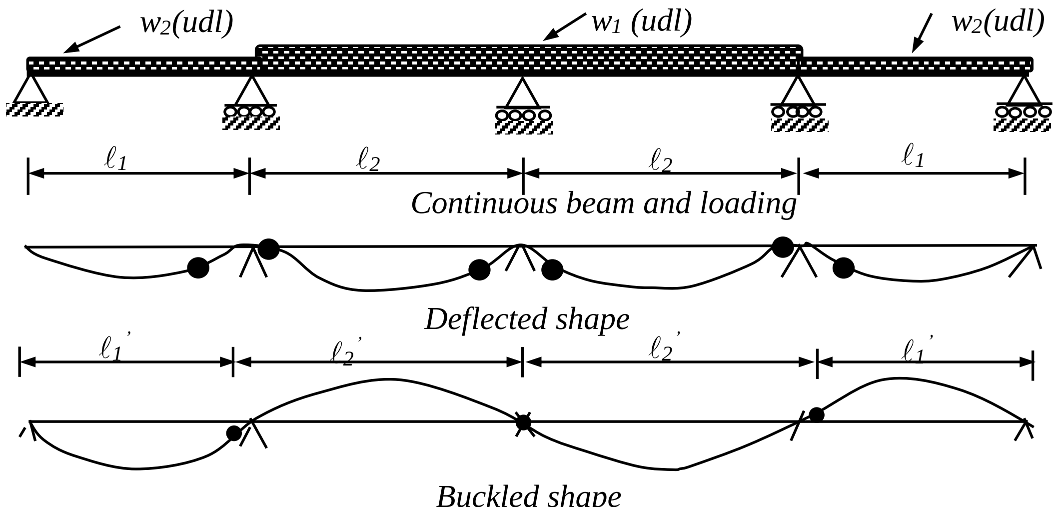


Fig. 3 Continuous beam – deflected shape and buckled shape

4.0 EFFECTIVE LATERAL RESTRAINT

Providing proper lateral bracing may increase the lateral stability of a beam. Lateral bracing may be either discrete (e.g. cross beams) or continuous (e.g. beam encased in concrete floors). The lateral buckling capacity of the beams with discrete bracing may be determined by using the methods described in a later Section. For the continuously restrained beams, assuming lateral deflection is completely prevented, design can be based on in-plane behaviour. It is important to note that in the hogging moment region of a continuous beam, if the compression flange (bottom flange) is not properly restrained, a form of lateral deflection with cross sectional distortion would occur.

4.1 Discrete bracing

In order to determine the behaviour of discrete braces, consider a simply supported beam provided with a single lateral support of stiffness K_b at the centroid, as shown in Fig. 4.

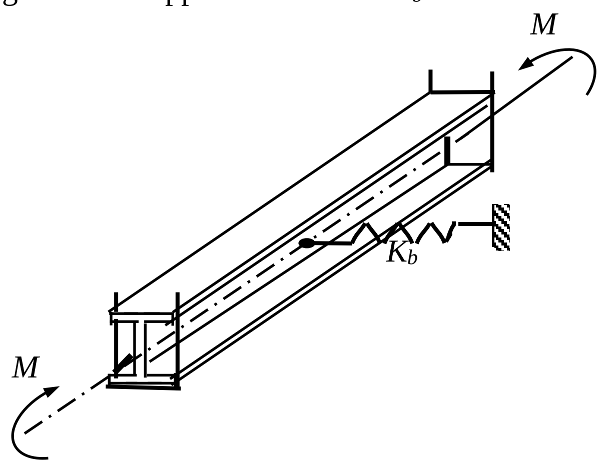


Fig. 4 Beam with single lateral support

The relationship between K_b and M_{cr} is shown in Fig. 5.

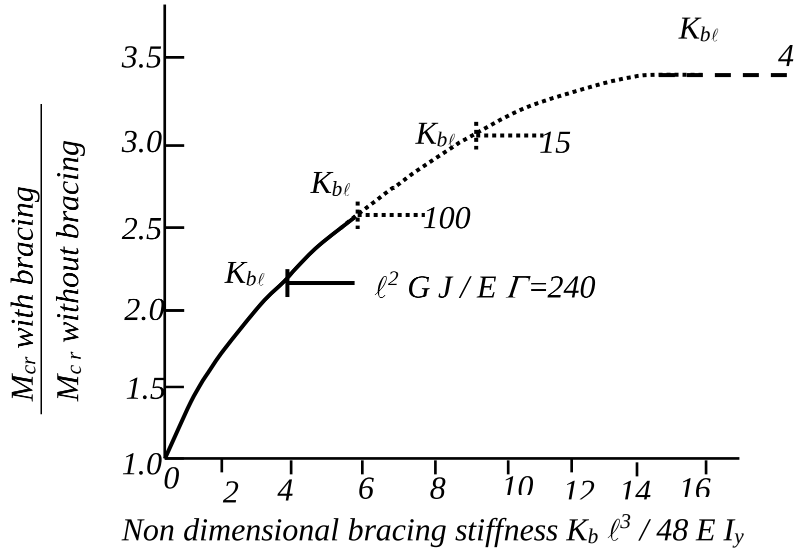


Fig. 5 Relationship between K_b and M_{cr}

It is seen that M_{cr} value increases with K_b, until K_b is equal to a limiting value of $K_{b\ell}$. The corresponding M_{cr} value is equal to the value of buckling for the two segments of the beam. M_{cr} value does not increase further as the buckling is now governed by the individual M_{cr} values of the two segments.

Generally, even a light bracing has the ability to provide substantial increase in stability. There are several ways of arranging lateral bracing to improve stability. The limiting value of the lateral bracing stiffness, $K_{b\ell}$, is influenced by the following parameters.

- Level of attachment of the brace to the beam i.e. top or bottom flange.
- The type of loading on the beam, notably the level of application of the transverse load
- Type of connection, whether capable of resisting lateral and torsional deformation
- The proportion of the beam.

Provision of bracing to tension flanges is not so effective as compression flange bracing. Bracing provided below the point of application of the transverse load would not be able to resist twisting and hence full capacity of the beam is not achieved. For the design of effective lateral bracing systems, the following two requirements are essential.

- Bracing should be of sufficient stiffness so that buckling occurs between the braces
- Lateral bracing should have sufficient strength to withstand the force transferred by the beam.

A general rule is that lateral bracing can be considered as fully effective if the stiffness of the bracing system is at least 25 times the lateral stiffness of the member to be braced.

Provisions in BS 5950 stipulate that adequate lateral and torsional restraints are provided if they are capable of resisting 1) a lateral force of not less than 1% of the maximum factored force in the compression flange for lateral restraints, 2) and a couple with lever arm equal to the depth between centroid of flanges and a force not less than 1% of the maximum factored compression flange force.

4.2 Continuous restraint

In a framed building construction, the concrete floor provides an effective continuous lateral restraint to the beam. As a result, the beam may be designed using in-plane strength. A few examples of fully restrained beams are shown in Fig. 6.

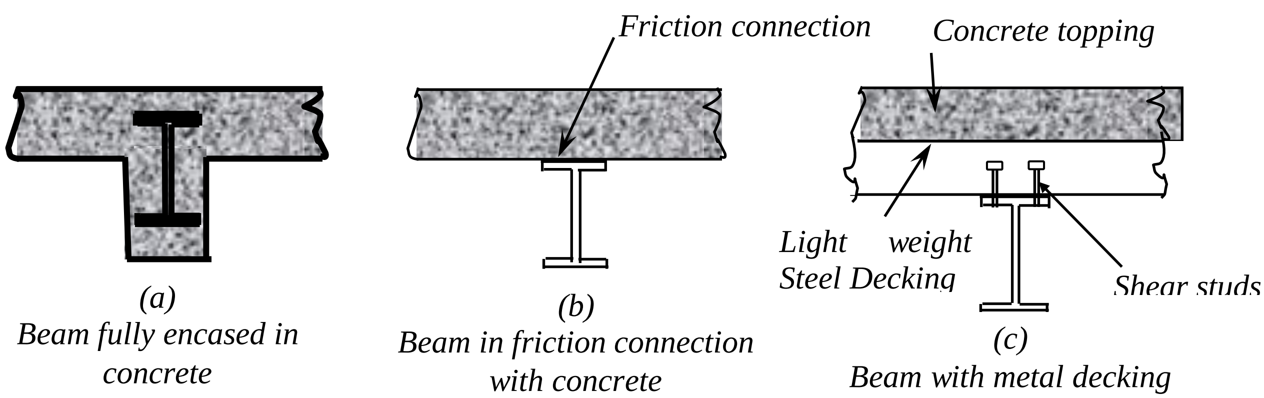


Fig. 6 Beams with continuous lateral restraint

The lateral restraint to the beam is effective only after the construction of the floor is completed. The beam will have to be temporarily braced after its erection till concreting is done and it has hardened.

For the case shown in Fig.6 (*a*), the beam is fully encased in concrete, and hence there will be no lateral buckling. In the arrangement shown in Fig.6 (*b*), the slab rests directly upon the beam, which is left unpainted. Full restraint is generally developed if the load transmitted and the area of contact between the slab and the beam are adequate to develop the needed restraint by friction and bond. For the case shown in Fig.6(*c*), the metal decking along with the concrete provides adequate bracing to the beam. However, the beam is susceptible to buckling before the placement of concrete due to the low shear stiffness of the sheeting. Shear studs are provided at the steel-concrete interface to enhance the shear resistance. The codal provisions require that for obtaining fully effective continuous lateral bracing, it must withstand not less than 1% of the maximum force in the compression flange.

5.0 BUCKLING OF MONOSYMMETRIC BEAMS

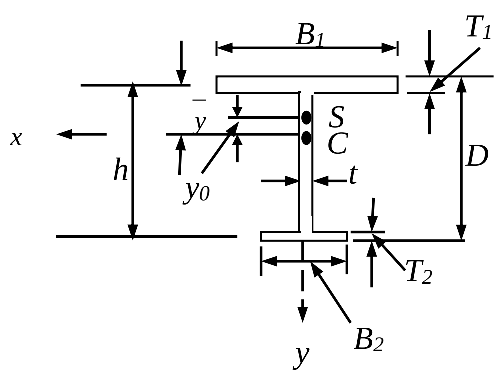
For beams symmetrical about the major axis only e.g. unequal flanged I- sections, the non-coincidence of the shear centre and the centroid complicates the torsional behaviour of the beam. The monosymmetric I-sections are generally more efficient in resisting loads provided the compressive flange stresses are taken by the larger flange.

When a monosymmetric beam is bent in its plane of symmetry and twisted, the longitudinal bending stresses exert a torque, which is similar to torsional buckling of short concentrically loaded compression members. The longitudinal stresses exert a torque, T_M given by

$$T_M = M_x\,\beta_x\,d\phi / dz \qquad (3)$$

$$\text{where } \beta_x = 1/I_x \int_A (x^2 y + y^3)\,dA - 2y_0 \qquad (4)$$

is the monosymmetry property of the cross section. Explicit expression for β_x for a monosymmetric I-section is given in Fig. 7.



$$h = D - (T_1 + T_2)/2$$

$$\bar{y} = \frac{B_2 T_2 h + (D - T_1 - T_2)(D - T_2)t/2}{B_1 T_1 + B_2 T_2 + (D - T_1 - T_2)t}$$

$$y_0 = \alpha h - \bar{y}$$

$$\alpha = \frac{1}{1 + (B_1/B_2)^3 (T_1/T_2)}$$

$$\Gamma = \alpha B_1^3 T_1 h^2 / 12$$

$$\beta_x = \frac{1}{I_x}\left\{ \begin{array}{l} (h - \bar{y})\left[B_2^3 T_2/12 + B_2 T_2 (h - \bar{y})^2 \right] \\ - \bar{y}\left[B_1^3 T_1/12 + B_1 T_1 \bar{y}^2 \right] + \left[(h - \bar{y} - T_2/2)^4 - (\bar{y} - T_1/2)^4 \right] t/4 \end{array} \right\}$$

Fig. 7 Properties of monosymmetric I-sections

The torque developed, T_m, changes the effective torsional rigidity of the section from GJ to $(GJ + M_x\,\beta_x)$. In doubly symmetric beams the torque exerted by the compressive bending stresses is completely balanced by the restoring torque due to the tensile stresses and therefore β_x is zero. In monosymmetric beams, there is an imbalance of torque due to larger stresses in the smaller flange, which is farther from the shear centre. Hence, when the smaller flange is in compression there is a reduction in the effective torsional rigidity;

$M_x \beta_x$ is negative and when the smaller flange is in tension $M_x\ \beta_x$ is positive. Thus, the principal effect of monosymmetry is that the buckling resistance is increased when the larger flange is in compression and decreased when the smaller flange is in compression. This effect is similar to the Wagner effect in columns. The value of critical moment for unequal flange I beam is given by.

$$M_c = \frac{\pi}{\ell}\sqrt{E\,I_y\,GJ}\left\{\sqrt{\left[1+\frac{\pi^2 E\Gamma}{GJ\ell^2}+\left(\frac{\pi\rho_m}{2}\right)^2\right]}+\frac{\pi\rho_m}{2}\right\} \tag{5}$$

Where $\rho_m = \dfrac{I_{yc}}{I_y}$ (6)

I_{yc} is the section minor axis second moment of area of the compression flange.

The monosymmetry property is approximated to

$$\beta_x = 0.9h\,(2\,\rho_m - 1)\,(1 - I_y^2 / I_x^2) \tag{7}$$

and the warping constant Γ by

$$\Gamma = \rho_m\,(1 - \rho_m)\,I_y\,h^2 \tag{8}$$

Very little is known of the effects of variations in the loading and the support conditions on the lateral stability of monosymmtric beams. However, from the available results, it is established that for top flange loading higher critical loads are always obtained when the larger flange is used as the compression flange. Similarly for bottom flange loading higher critical loads can be obtained when this is the larger flange. For Tee- sections β_x can be obtained by substituting the flange thickness T_1 or T_2 equal to zero; also for Tee sections the warping constant, Γ is zero.

6.0 BUCKLING OF NON-UNIFORM BEAMS

Non-uniform beams are often used in situations, where the strong axis bending moment varies along the length of the beam. They are found to be more efficient than beams of uniform sections in such situations. The non-uniformity in beams may be obtained in several ways. Rectangular sections generally have taper in their depths. I-beams may be tapered in their depths or flange widths; flange thickness is generally kept constant. However, steps in flange width or thickness are also common.

Tapering of narrow rectangular beams will produce considerable reduction in minor axis flexural rigidity, EI_y, and torsional rigidity, GJ; consequently, they have low resistance to lateral torsional buckling. Reduction of depth in I-beams does not affect EI_y, and has only marginal effect on GJ. But warping rigidity, $E\Gamma$, is considerably reduced. Since the contribution of warping rigidity to buckling resistance is marginal, depth reduction does not influence significantly the lateral buckling resistance of I beams. However, reduction in flange width causes large reduction in GJ, EI_y and $E\Gamma$. Similarly, reduction in flange thickness will also produce large reduction in EI_y, $E\Gamma$, and GJ in that order. For small degrees of taper there is little difference between width-tapered beams and thickness tapered beams. But for highly tapered beams, the critical loads of thickness tapered ones are higher. Thus, the buckling resistance varies considerably with change in the flange geometry.

Based on the analysis of a number of beams of different cross sections with a variety of loading and support conditions, the elastic critical load for a tapered beam may be determined approximately by applying a reduction factor r to the elastic critical load for an equivalent uniform beam possessing the properties of the cross section at the point of maximum moment

$$r = \frac{7+\gamma}{5+3\gamma} \tag{9}$$

$$\gamma = \frac{S_{x0}}{S_{x1}}\left[\left(\frac{D_1}{D_0}\right)^3\left(\frac{B_0}{B_1}\right)^3\left(\frac{T_0}{T_1}\right)^2\right] \tag{10}$$

S_x = section modulus.

T = flange thickness.

D = depth of the section.

B =flange width.

Subscripts 0 and 1 relate to the points of maximum and minimum moment respectively.

For the design of non-uniform sections, BS 5950 provides a simple method, in which the properties where the moment is maximum may be used and the value of n is suitably adjusted. The value of n is given by

$$n = 1.5 - 0.5\, A_{sm} / A_{lm} \geq 1.0 \tag{11}$$

Where A_{sm} and A_{lm} are flange areas at the points of the smallest and largest moment, and m = 1.0

7.0 BEAMS OF UNSYMMETRICAL SECTIONS

The theory of lateral buckling of beams developed so far is applicable only to doubly symmetrical cross sections having uniform properties throughout its length. Many lateral buckling problems encountered in design practice belong to this category. However, cases may arise where the symmetry property of the section may not be available. Such cases are described briefly in this Section.

The basic theory can also be applied to sections symmetrical about minor axis only e.g. Channels and Z-sections. In this section, the shear centre is situated in the axis of symmetry although not at the same point as the centroid. In the case of channel and *Z* sections, instability occurs only if the loading produces pure major axis bending. The criterion is satisfied for the two sections if: (1) for the channel section, the load must act through the shear centre Fig.8 (a) and, (2) for *Z*-section in a direction normal to the horizontal principal plane [Fig.8 (b)].

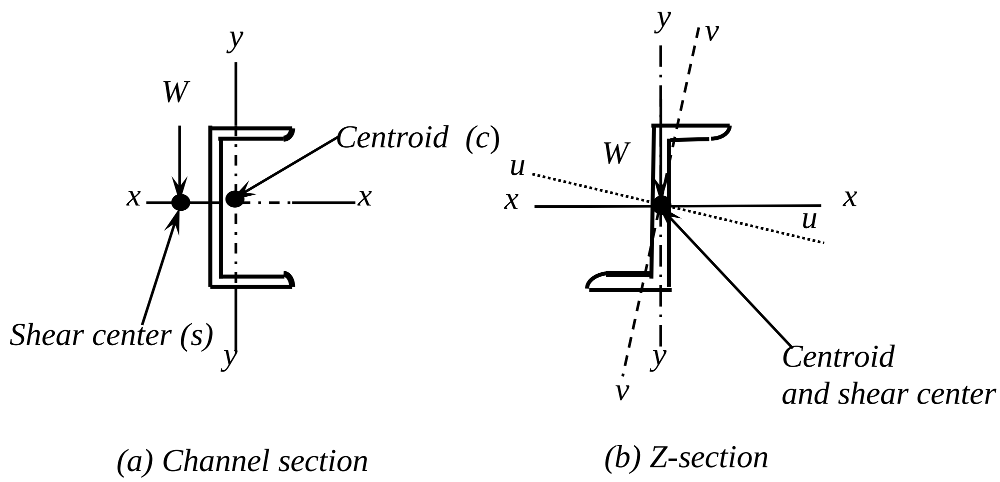


Fig. 8 Loading through shear centre

If these conditions are satisfied, M_{cr} of these sections can be obtained using their properties and the theoretical equation. The warping constant Γ for the sections are:

$$\Gamma = \frac{T B^3 h}{12}\left[\frac{3BT + 2ht}{6BT + ht}\right] \text{ for a channel} \tag{12}$$

$$\Gamma = \frac{B^3 h^2}{12(2Bth)^2}\left[2t\,(B^2 + Bh + h^2) + 3t\,Bh\right] \text{for a Z}-\text{section} \tag{13}$$

where,

h = distance between flange centroids
t = thickness of web
B = total width of flange
T = flange thickness.

It is very difficult to obtain such loading arrangements so as to satisfy the restrictions mentioned above. In such cases, the behaviour may not be one of lateral stability; instead a combination of bending and twisting or bi-axial bending.

For sections, which have symmetry about minor axis only, their shear centre does not coincide with the centroid. This results in complicated torsional behaviour and theoretical predictions are not applicable. Fig. 9 shows the instability behaviour of sections with flanges of varying sizes and positions (top or bottom). It can be seen from the figure that sections with flange in the compression region are more advantageous.

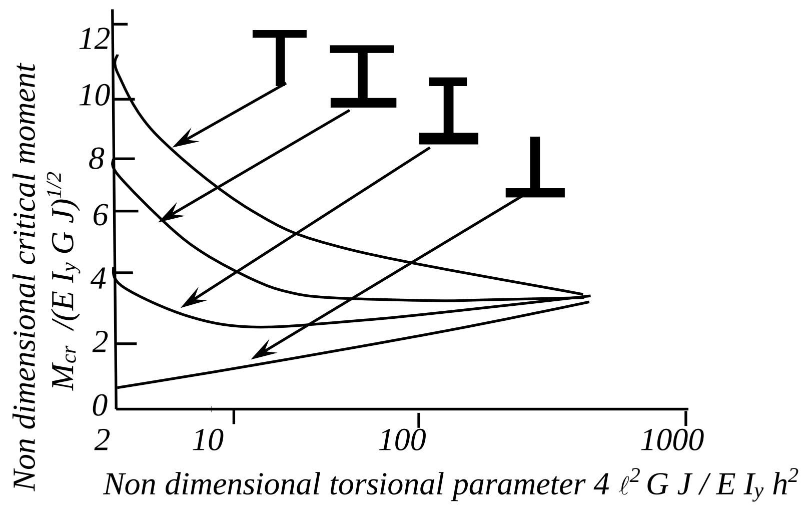


Fig. 9 Effect of flange position and proportion on lateral stability

While considering the case of tapered beams, it has been established, based on lateral stability studies, that variation of the flange properties can cause large changes in the lateral buckling capacity of the beam, whereas tapering of depth has insignificant influence on the buckling capacity.

8.0 SUMMARY

In this chapter, lateral torsional buckling of some practical cases of beams have been explained. It is pointed out that for cantilever beams the type of restraint provided at fixed -end plays a significant role in their buckling capacities. Torsional restraint of the cantilever beam has been found to be more beneficial than lateral restraint. In the case of beams with equally spaced and loaded cross beams the critical moment of the main beam and the associated buckling mode will depend on the spacing of the cross beams. Requirements for effective lateral restraint have been presented. Continuous restraint provided by concrete floors to beams in composite constructions of buildings is discussed. As discussed in an earlier chapter, the local buckling effects should be taken into account by satisfying the minimum requirements of the member cross-section. Cases of monosymmetric beams and non- uniform beams are also briefly explained. Finally cases of beams with un-symmetric sections are discussed and concluded that beams with flanges in the compression zone are more advantageous from the point of view of lateral torsional buckling.

9.0 REFERENCES

1. Trahair N.S., ‘The behaviour and design of steel structures’, Chapman and Hall London, 1977

2. Kirby P.A. and Nethercot D.A.,’Design for structural stability’, Granada Publishing, London, 1979

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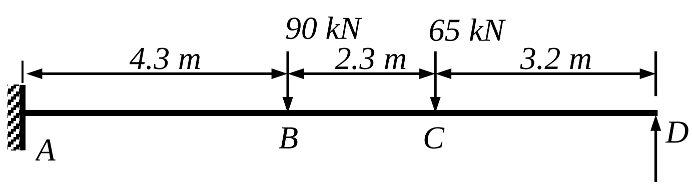
Problem I :

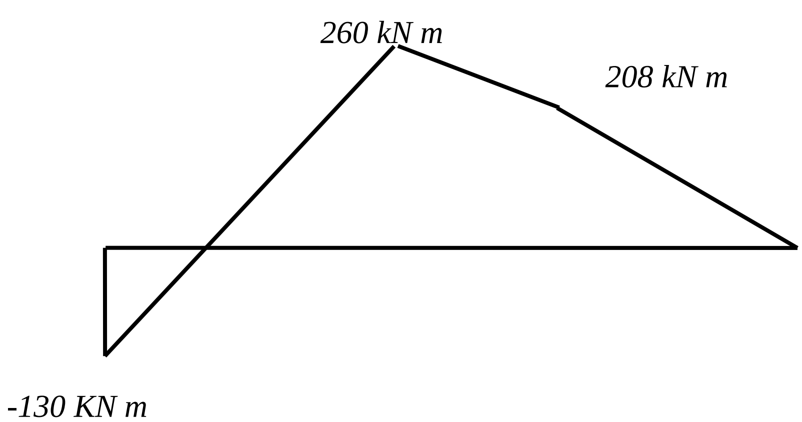
A propped cantilever has a span of 9.8 m. it is loaded by cross beams at 4.3 m and 6.6 m from its left hand end. The ends of the beam and the loaded points are assumed to be fully braced laterally and torsionally.



The loads as given are factored.

Design a suitable section for the beam:

The bending moment diagram of the beam is as shown below.



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Section properties of ISMB 450 are :

Depth D = 450 mm.

Width B = 150 mm.

Web thickness t = 9.4 mm.

Flange thickness T = 17.4 mm.

Depth between fillets, d = 379.2 mm.

Radius of gyration about

Minor axis, $r_y = 30.1$ *mm.*

Plastic modulus about
major axis, $S_x = 1512.78\ cm^3$.

Assume $f_y = 250\ N / mm$, $E = 200000\ N / mm^2$, $\gamma_m = 1.15$

$P_y = f_y / \gamma_m = 250 / 1.15 = 217.4\ N / mm^2$

(1) ***Type of section***

(i) flange criterion:

$$b = \frac{B}{2} = \frac{150}{2} = 75 \text{ mm}$$

$$\frac{b}{T} = \frac{75}{17.4} = 4.31$$

$$\frac{b}{T} < 8.92\varepsilon, \text{ where } \varepsilon = \sqrt{\frac{250}{f_y}}$$

Hence o.k.

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(ii) *Web criterion:*

$$\frac{d}{t} = \frac{379.2}{9.4} = 40.3$$

$$\frac{d}{t} < 82.95\,\varepsilon$$

Hence o.k.

Since

$\frac{b}{t} < 8.92\varepsilon$ *and* $\frac{d}{t} < 82.95\ \varepsilon$ *, the section is classified as " plastic"*

Check for moment capacity:

$$M_C = S_x * p_y$$

$$= \frac{1512.78 * 217.4}{1000} = 328.87\ kN\,m$$

Maximum applied moment $= 260\ KN - m < 328.87\ KN\ \ m$

Hence o.k.

(iii) *Lateral torsional buckling:*

The beam length AB, BC and CD will be treated separately using the equivalent uniform method.

Effective lengths:

$$\ell_{AB} = 4.3\ m.$$

$$\ell_{BC} = 2.3\ m.$$

$$\ell_{CD} = 3.2\ m.$$

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Length L_{AB}:

The equivalent uniform moment should be less than the lateral torsional buckling resistance moment

$$\overline{M} \le M_b$$

where,

$\overline{M}$ *is equivalent uniform moment*

M_b *is lateral torsional buckling resistance moment*

$$\overline{M} = m\, M_A$$

where,

M_A *is the maximum moment in the member*

m *is the equivalent uniform moment factor*

To determine 'm' :

$$m = 0.57 + 0.33\,\beta + 0.1\,\beta^2 \not< 0.43\text{ , where } \beta = \frac{M_{min}}{M_{max}}$$

$$\beta = -\frac{130}{260} = -0.5\text{, then } m = 0.43$$

$$\overline{M} = 0.43 * 260 = 112\ kN\,m$$

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Length L_{BC}:

$$\beta = \frac{208}{260} = 0.8\,; \text{ Then } m = 0.9$$

$$\overline{M} = 0.9 * 260 = 234 \text{ kN m}$$

length L_{CD}:

$$\beta = 0;\ m = 0.57.\ \overline{M} = 0.57 * 208 = 119 \text{ kN m}$$

For the purpose of determining the governing values, all the three segments are checked separately.

$$\lambda = \frac{\ell_{AB}}{r_y} = \frac{4.3 * 1000}{30.1} = 142.86$$

$$x = \frac{D}{T} = 25.86$$

$$\frac{\lambda}{x} = \frac{142.86}{25.86} = 5.52$$

$$v = 0.79$$

$$\lambda_{LT} = n\,u\,v\,\lambda\ ;\quad u = 0.9\ ,\ n = 1.0$$

$$= 1.0 * 0.9 * 0.79 * 142.86$$

$$= 101.57$$

Bending strength, $P_b = 117$ *Mpa* (for $\lambda_{LT} = 101.57$)

Buckling resistance moment

$$M_b = \frac{1512.78 * 117}{1000} = 177 > 112 \text{ kN m}$$

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Length ℓ_{AB} is safe against lateral torsional buckling

Length ℓ_{BC} :

$$\lambda = \frac{\ell_{BC}}{r_y} = \frac{2300}{30.1} = 76.41$$

$$x = \frac{D}{T} = 25.86$$

$$\frac{\lambda}{x} = \frac{76.41}{25.86} = 2.95$$

$$v = 0.91$$

$$\lambda_{LT} = 1.0 * 0.9 * 0.91 * 76.41 = 62.58$$

Bending strength , p_b = 190 Mpa

Buckling resistance moment M_b

$$= \frac{1512.78 * 190}{1000} = 287.43 > 234 \ kN\,m$$

Length ℓ_{BC} is safe against lateral torsional buckling

Length ℓ_{CD} :

$$\lambda = \frac{\ell_{CD}}{r_y} = \frac{3.2 * 1000}{30.1} = 106.31$$

$$x = \frac{D}{T} = 25.86$$

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$$\frac{\lambda}{x} = \frac{106.31}{25.86} = 4.11$$

$$v = 0.70$$

$$\lambda_{LT} = 1 * 0.9 * 0.7 * 106.31 = 66.97$$

bending strength, $p_b = 174$ *Mpa*

$$\text{bending resistance moment} = \frac{174 * 1512.78}{1000}$$

$$= 263.22 > 119 \text{ kN m}$$

Length L_{CD} *is safe against lateral torsional buckling.*

There fore the section chosen 'ISMB 450' is o.k.

The shortest segment BC, which has the most severe pattern of moments controls the design.