38 MULTI-STOREY BUILDINGS - II

1.0 INTRODUCTION

Modern design offices are generally equipped with a wide variety of structural analysis software programs, invariably based on the stiffness matrix method. These Finite Element Analysis packages such as MSC/NASTRAN, SAP - 90, STAAD etc., give more accurate results compared with approximate methods, but they involve significant computational effort and therefore cost. They are generally preferred for complex structures. The importance of approximate hand methods for the analysis of forces and deflections in multi-storeyed frames can not be ignored; they have served the Structural Engineer well for many decades and are still useful for preliminary analysis and checking. This chapter describes various approximate methods to analyse simple and rigid multi-storeyed frames.

2.0 BRACED FRAMES - METHODS OF ANALYSIS FOR LATERAL LOADS

In this section, simple hand methods for the analysis of statically determinate or certain low-redundant braced structures is reviewed.

2.1 Member Force Analysis

Analysis of the forces in a statically determinate triangulated braced frame can be made by the method of sections. For instance, consider a typical single-diagonal braced pin-jointed panel as shown in Fig. 1. When this bent is subjected to an external shear Q_i in *i*-th storey and external moments M_i and M_{i-1} at floors i and $i-1$, respectively, the force in the brace can be found by considering the horizontal equilibrium of the free body above section *XX*, thus,

$$F_{BC} \cos\theta = Q_i$$

Hence,

$$F_{BC} = Q_i / \cos\theta$$

The axial force F_{BD} in the column *BD* is found by considering moment equilibrium of the upper free body about *C*, thus

$$F_{BD} * \ell = M_{i-1}$$

Hence,

$$F_{BD} = M_{i-1} / \ell$$

Similarly the force F_{AC} in column AC is obtained from the moment equilibrium of the upper free body about B. It is given by

$F_{AC} = M_i / \ell$

This procedure can be repeated for the members in each storey of the frame. The member forces in more complex braced frames such as knee-braced, X-braced and K-braced frames can also be obtained by taking horizontal sections.

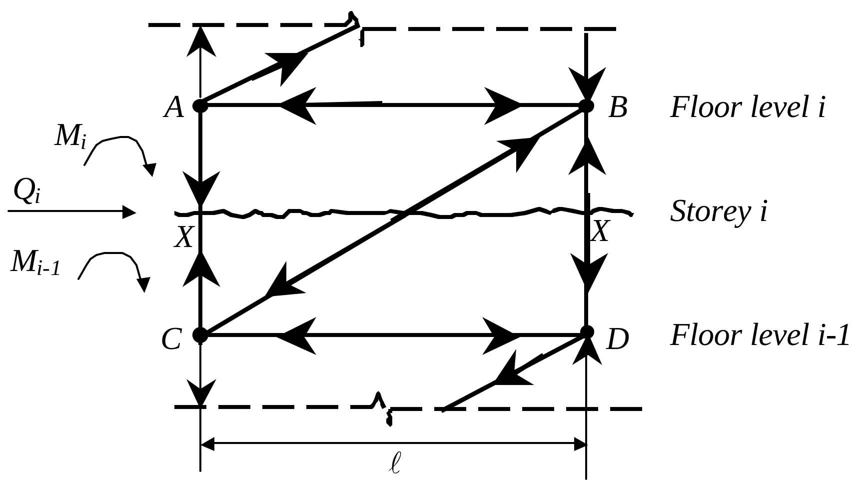


Fig. 1 Single diagonal braced panel

2.2 Drift Analysis

Drift in building frames is a result of flexural and shear mode contributions, due to the column axial deformations and to the diagonal and girder deformations, respectively. In low rise braced structures, the shear mode displacements are the most significant and, will largely determine the lateral stiffness of the structure. In medium to high rise structures, the higher axial forces and deformations in the columns, and the accumulation of their effects over a greater height, cause the flexural component of displacement to become dominant.

The storey drift in a braced frame reaches a maximum value at or close to the top of the structure and is strongly influenced by the flexural component of deflection. This is because the inclination of the structure caused by the flexural component accumulates up the structure, while the storey shear component diminishes toward the top.

Hand analysis for drift allows the drift contributions of the individual frame members to be seen, thereby providing guidance as to which members should be increased in size to effectively reduce an excessive total drift or storey drift. The following section explains a method for hand evaluation of drift.

2.2.1 Virtual work drift analysis

In this method, a force analysis of the structure is carried out for design lateral loads in order to determine the axial force P_j in each member j and the bending moment M_{xj} at sections x along those members subjected to bending [See Fig. 2(*a*)]. A second force analysis is then carried out with the structure subjected to only a unit imaginary or "dummy" lateral load at the level N whose drift is required [Fig. 2(*b*)] to give the axial force p_{jN} , and moment m_{xjN} at section x in the bending members. The resulting horizontal deflection at N is then given by

$$D_N = S\, p_{jN}\left(\frac{P\ell}{EA}\right)_j + S\int_0^{\ell_j} m_{xjN}\left(\frac{M_x}{EI}\right)_j dx \qquad (1)$$

where, ℓ_j - length of member j
A_j - sectional area of member j
E - elastic modulus
I_j - moment of inertia of member j.

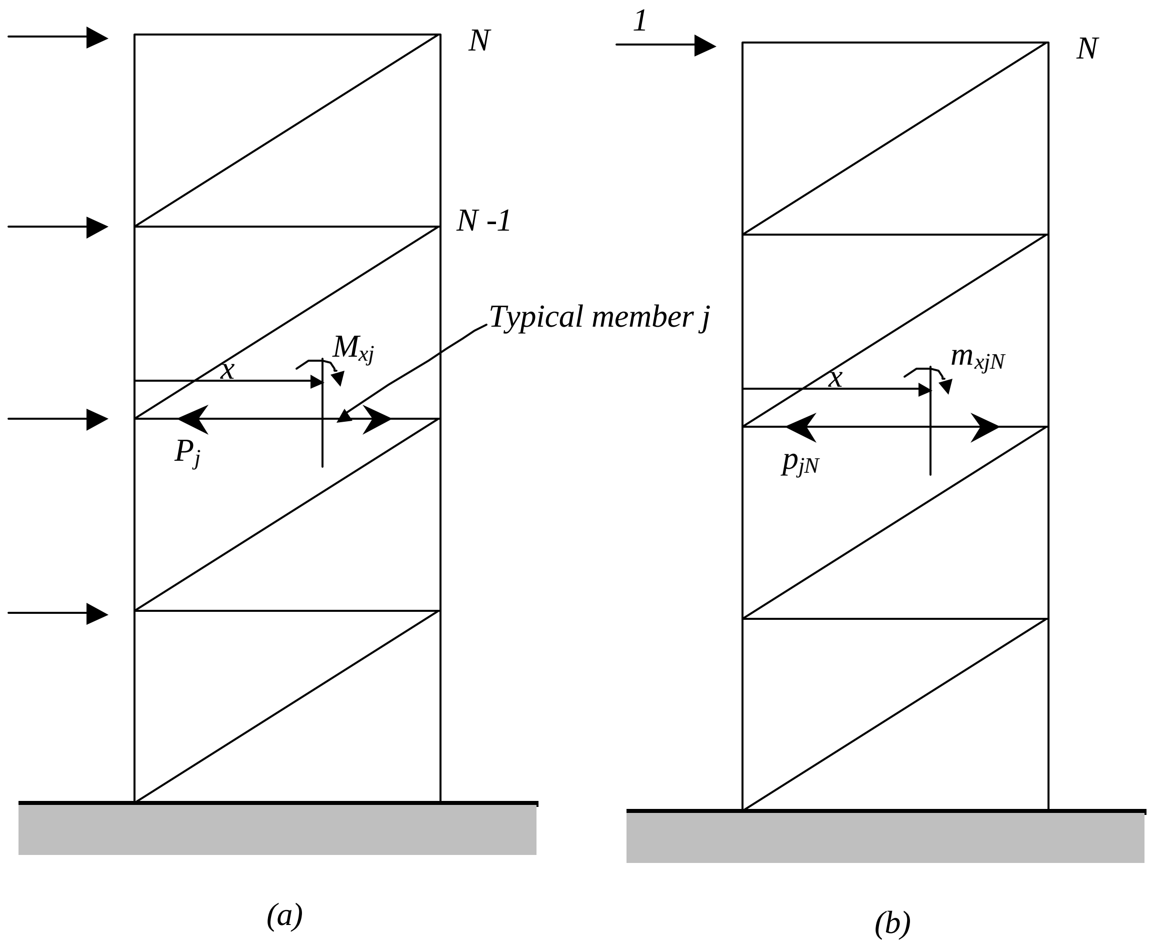


Fig. 2 Member forces in a typical braced frame

This method is exact and can easily be systematised by tabulation. A good assessment of the deflected configuration, the total drift, and the storey drifts can be obtained by plotting the deflection diagram from the deflections at just three or four equally spaced points up the height of the structure. It requires one design load force analysis and three or four "dummy" unit load analyses.

3.0 ANALYSIS OF FRAMES WITH MOMENT-RESISTING JOINTS FOR LATERAL LOADS

Multi-storey building frames subjected to lateral loads are statically indeterminate and exact analysis by hand calculation takes much time and effort. Using simplifying assumptions, approximate analyses of these frames yield good estimate of member forces in the frame, which can be used for checking the member sizes. The following methods can be employed for lateral load analysis of rigidly jointed frames.

- The Portal method.
- The Cantilever method
- The Factor method

The portal method and the cantilever method yield good results only when the height of a building is approximately more than five times its least lateral dimension. Either classical techniques such as slope deflection or moment distribution methods or computer methods using stiffness or flexibility matrices can be used if a more exact result is desired.

3.1 The Portal Method

This method is satisfactory for buildings up to *25* stories, hence is the most commonly used approximate method for analysing tall buildings. The following are the simplifying assumptions made in the portal method:

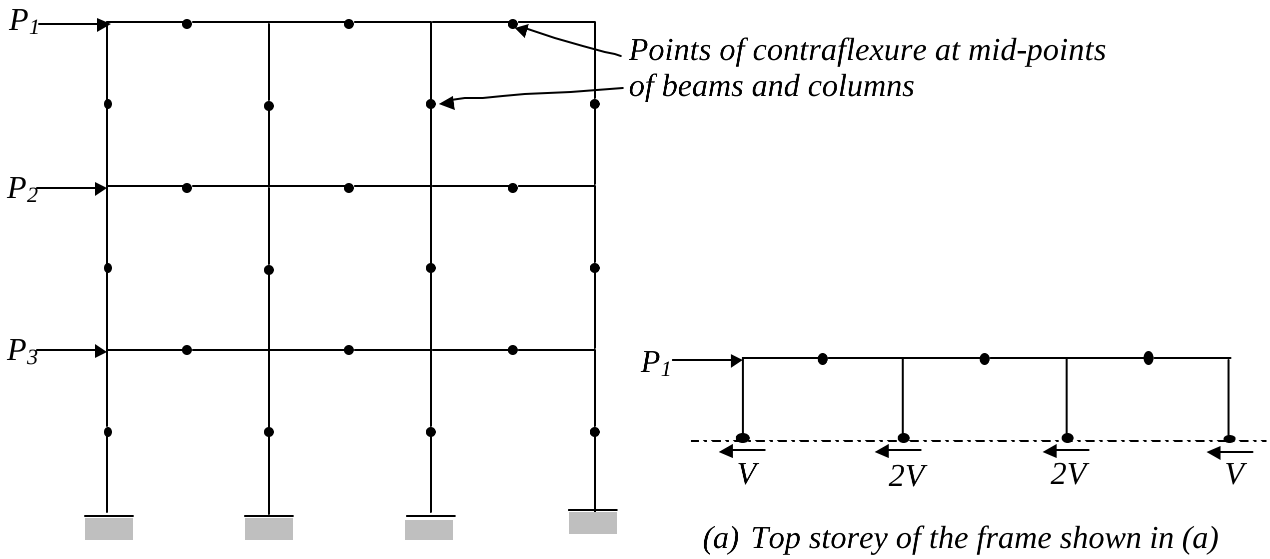
1. A point of contraflexure occurs at the centre of each beam.
2. A point of contraflexure occurs at the centre of each column.
3. The total horizontal shear at each storey is distributed between the columns of that storey in such a way that each interior column carries twice the shear carried by each exterior column.

The above assumptions convert the indeterminate multi-storey frame to a determinate structure. The steps involved in the analysis of the frame are detailed below:

1. The horizontal shears on each level are distributed between the columns of that floor according to assumption (3).
2. The moment in each column is equal to the column shear multiplied by half the column height according to assumption (2).
3. The girder moments are determined by applying moment equilibrium equation to the joints: by noting that the sum of the girder moments at any joint equals the sum of the

column moments at that joint. These calculations are easily made by starting at the upper left joint and working joint by joint across to the right end.

4. The shear in each girder is equal to its moment divided by half the girder length. This is according to assumption (1).
5. Finally, the column axial forces are determined by summing up the beam shears and other axial forces at each joint. These calculations again are easily made by working from left to right and from the top floor down.



(a) A multi-storey frame showing points of contraflexure

(a) Top storey of the frame shown in (a) showing horizontal shear distribution in the columns of top storey

Fig. 3 A multi-storey frame subjected to wind loading

Assumptions of the Portal method of analysis are diagrammatically shown in Fig. 3 and method of analysis is illustrated in worked example - *1*

3.2 The Cantilever Method

This method gives good results for high-narrow buildings compared to those from the Portal method and it may be used satisfactorily for buildings of *25* to *35* storeys tall. It is not as popular as the portal method.

The simplifying assumptions made in the cantilever method are:

1. A point of contraflexure occurs at the centre of each beam
2. A point of contraflexure occurs at the centre of each column.
3. The axial force in each column of a storey is proportional to the horizontal distance of the column from the centre of gravity of all the columns of the storey under consideration.

The steps involved in the application of this method are:

1. The centre of gravity of columns is located by taking moment of areas of all the columns and dividing by sum of the areas of columns.
2. A lateral force P acting at the top storey of building frame is shown in Fig. 4(*a*). The axial forces in the columns are represented by F_1, F_2, F_3 and F_4 and the columns are at a distance of x_1, x_2 , x_3 and x_4 from the centroidal axis respectively as shown in Fig. 4(*b*).

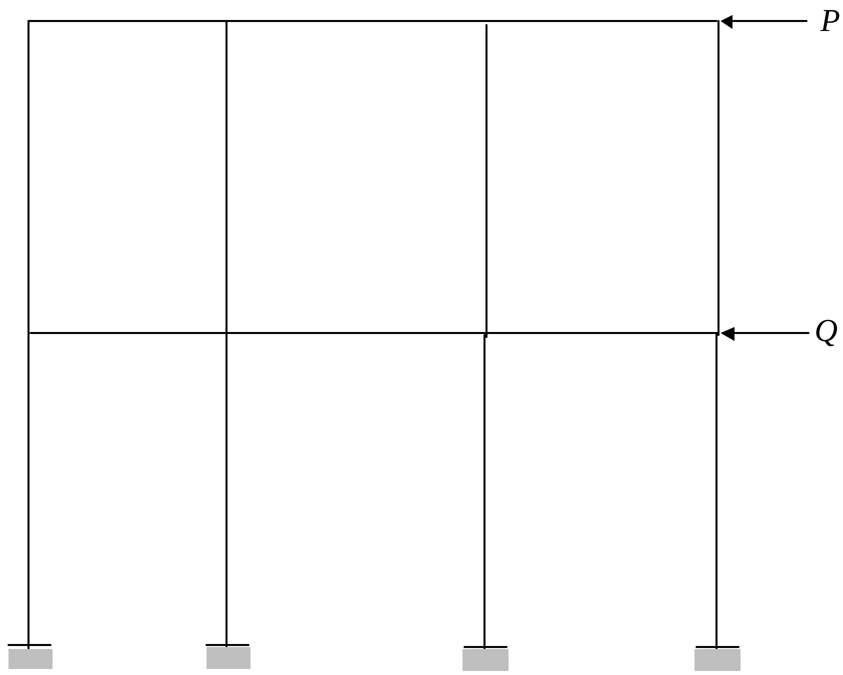


Fig. 4(a) Typical frame

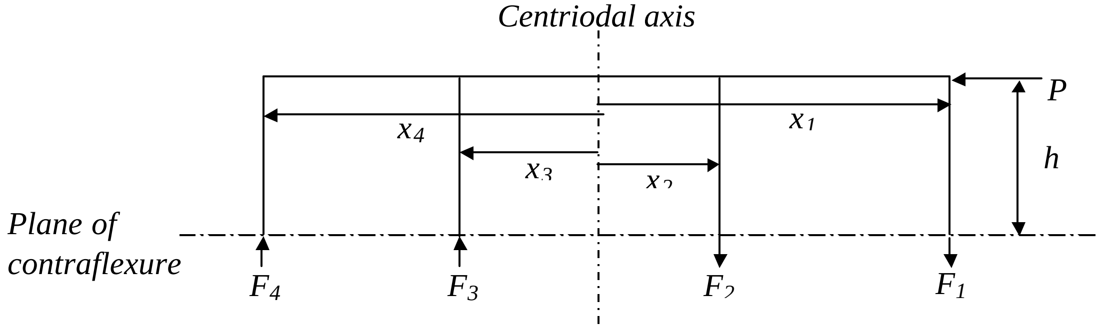


Fig. 4(b) Top storey of the above frame above plane of contraflexure

By taking the moments about the centre of gravity of columns of the storey,

$$P\,h - F_1x_1 - F_2x_2 - F_3x_3 - F_4x_4 = 0$$

The axial force in one column may be assumed as F and the axial forces of remaining columns can be expressed in terms of F using assumption (3).

3. The beam shears are determined joint by joint from the column axial forces.
4. The beam moments are determined by multiplying the shear in the beam by half the span of beam according to assumption (1).
5. The column moments are found joint by joint from the beam moments.
6. The column shears are obtained by dividing the column moments by the half-column heights using assumption (2)

The cantilever method analysis is illustrated in worked example - *2*.

3.3 The Factor Method

The factor method is more accurate than either the portal method or the cantilever method. The portal method and cantilever method depend on assumed location of hinges and column shears whereas the factor method is based on assumptions regarding the elastic action of the structure. For the application of Factor method, the relative stiffness ($k = I/\ell$), for each beam and column should be known or assumed, where, I is the moment of inertia of cross section and ℓ is the length of the member.

The application of the factor method involves the following steps:

1. The girder factor g, is determined for each joint from the following expression.

$$g = \frac{\sum k_c}{\sum k}$$

where, $S\,k_c$ - Sum of relative stiffnesses of the column members meeting at that joint.

$S\,k$ - Sum of relative stiffnesses of all the members meeting at that joint.

Each value of girder factor is written at the near end of the girder meeting at the joint.

2. The column factor c, is found for each joint from the following expression

$$c = 1-g$$

Each value of column factor c is written at the near end of each column meeting at the joint. The column factor for the column fixed at the base is one.

At each end of every member, there will be factors from step 1 or step 2. To these factors, half the values of those at the other end of the same member are added.

3. The sum obtained as per step 2 is multiplied by the relative stiffness of the respective members. This product is termed as column moment factor C, for the columns and the girder moment factor G, for girders.

4. *Calculation of column end moments for a typical member ij* - The column moment factors [*C* values] give approximate relative values of column end moments. The sum of column end moments is equal to horizontal shear of the storey multiplied by storey height. Column end moments are evaluated by using the following equation,

$$M_{ij} = C_{ij} A$$

where, M_{ij} - moment at end *i* of the *ij* column
C_{ij} - column moment factor at end *i* of column *ij*
A - storey constant given by

$$A = \left(\frac{Total\ horizontal\ shear\ of\ storey \times Height\ of\ the\ storey}{Sum\ of\ the\ column\ end\ moment\ factors\ of\ the\ storey} \right)$$

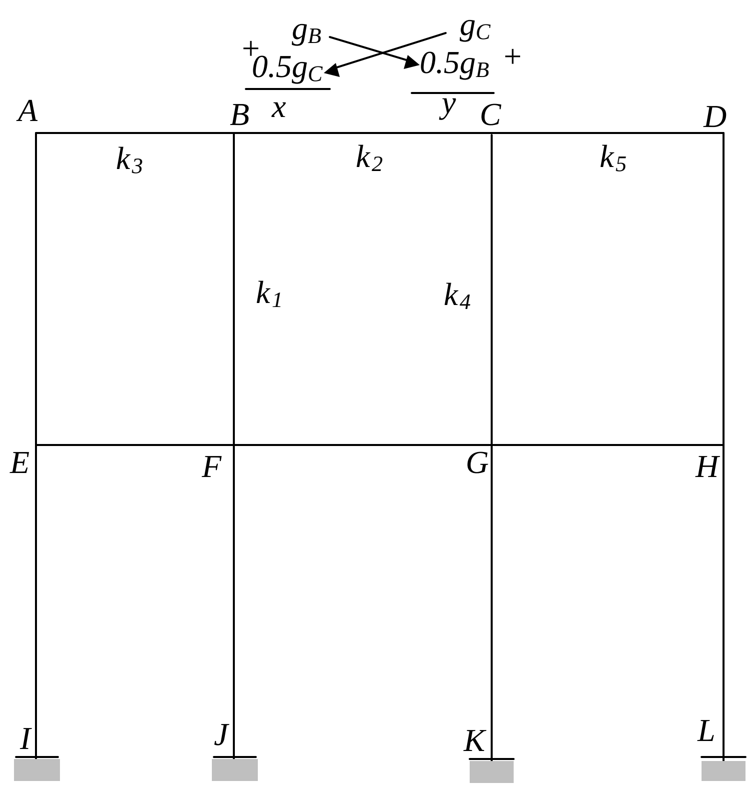


Fig. 5 Typical frame

5. *Calculation of beam end moments* - The girder moment factors [*G* values] give the approximate relative beam end moments. The sum of beam end moments at a joint is equal to the sum of column end moments at that joint. Beam end moments can be worked out by using following equation,

$$M_{ij} = G_{ij} B$$

where, M_{ij} - moment at end *i* of the *ij* beam

G_{ij} - girder moment factor at end i of beam ij
B - joint constant given by

$$B = \left(\frac{Sum\ of\ column\ moments\ at\ the\ joint}{Sum\ of\ the\ girder\ end\ moment\ factors\ of\ that\ joint} \right)$$

Illustration of calculation of G values:

Consider the joints B and C in the frame shown in Fig. 5.

Joint B: $g_B = k_1 /(k_1 + k_2 + k_3)$

$c_B = 1 - g_B$

Joint C: $g_C = k_4 /(k_2 + k_4 + k_5)$

$c_C = 1 - g_C$

As shown in Fig. 5, we should obtain values like x and y at each end of the beam and column. Thereafter we multiply them with respective k values to get the column or girder moment factors. Here, $G_{BC} = x\ k_2$ and $G_{CB} = y\ k_2$. Similarly we calculate all other moment factors. The detailed factor method of analysis is illustrated in the worked example -*3*.

4.0 ANALYSIS OF BUILDINGS FOR GRAVITY LOADS

As discussed in previous chapter, building frames may be of three types, namely, simple framing, semi-rigid framing and rigid framing. Generally, the beams and girders of upper floors may very well be designed on the basis of simple beam moments, while those of lower floors may be designed as continuous members with moment resisting connections.

4.1 Simple Framing

If a simple framing is used, the design of beams is quite simple because they can be considered as simply supported. In such cases, shears and moments can be determined by statics. The gravity loads applied to the columns are relatively easy to estimate, but the column moments may be a little more difficult to find out. The column moments occur due to uneven distribution and unequal magnitude of live load. If the beam reactions are equal on each side of interior column, then there will be no column moment. If the reactions are unequal, the moment produced in the column will be equal to the difference between reactions multiplied by eccentricity of the beam reaction with respect to column centre line.

4.2 Semi Rigid Framing

The analysis of semi-rigid building frames is complex. The semi-rigid frames are designed by using special techniques developed based on experimental evidence. This will be discussed in a later chapter.

4.3 Rigid Framing

Rigid frame buildings are analysed by one of the approximate methods to make an estimate of member sizes before going to exact methods such as slope-deflection or moment-distribution method. If the ends of each girder are assumed to be completely fixed, the bending moments due to uniform loads are as shown in full lines of Fig. 6(*a*). If the ends of beam are connected by simple connection, then the moment diagram for uniformly distributed load is shown in Fig. 6(*b*). In reality, a moment somewhere between the two extremes will occur which is represented by dotted line in Fig. 6(*a*). A reasonable procedure is to assume fixed end moment in the range of $w\ell^2/10$, where ℓ is clear span and w is magnitude of uniformly distributed load.

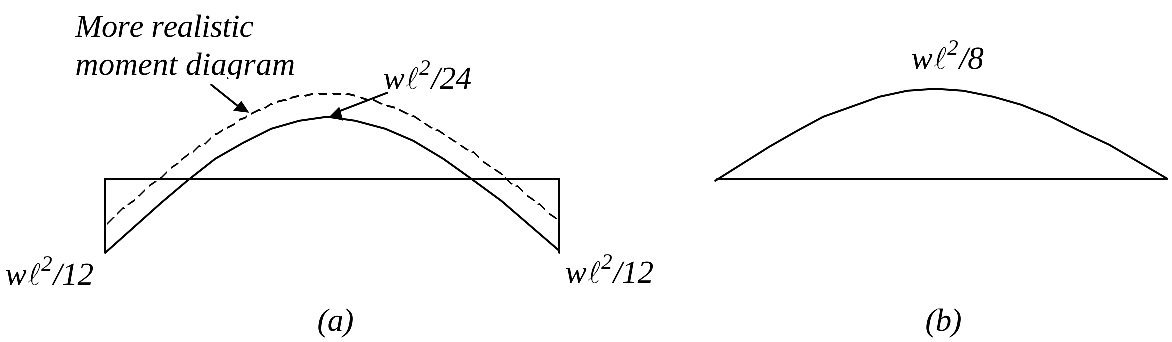


Fig. 6 (a) Fixed beam (b) Simply - supported beam bending moment diagrams

4.3.1 Analysis of structural frames for gravity loads - (according to IS: 456 - 1978)

The following assumptions are made for arrangement of live load for the analysis of frames:

a) Consideration is limited to combination of:
   1. Design dead load on all spans with full design live load on two adjacent spans and
   2. Design dead load on all spans with full design live load on alternate spans.

b) When design live load does not exceed three-fourths of the design dead load, the load arrangement of design dead load and design live load on all the spans can be used.

Unless more exact estimates are made, for beams of uniform cross-section which support substantially uniformly distributed loads over three or more spans which do not differ by more than *15%* of the longest, the bending moments and shear forces used for design is obtained using the coefficients given in Table 1 and Table 2 respectively. For moments at supports where two unequal spans meet or in cases where the spans are not equally loaded, the average of the two values for the hogging moment at the support may be used for design.

Where coefficients given in Table 1 are used for calculation of bending moments, redistribution of moments is not permitted.

Table 1: Bending moment coefficients

TYPE OF LOAD	**SPAN MOMENTS**		**SUPPORT MOMENTS**	
	Near middle span	At middle of interior span	At support next to the end support	At other interior supports
Dead load + Imposed load (fixed)	+ 1/12	+1/24	- 1/10	- 1/12
Imposed load (not fixed)	+1/10	+1/12	- 1/9	- 1/9
For obtaining the bending moment, the coefficient is multiplied by the total design load and effective span.				

Table 2: Shear force coefficients

TYPE OF LOAD	**At end support**	**At support next to the end support**		**At all other interior supports**
		Outer side	Inner side	
Dead load + Imposed load(fixed)	0.40	0.60	0.55	0.50
Imposed load(not fixed)	0.45	0.60	0.60	0.60
For obtaining the shear force, the coefficient is multiplied by the total design load				

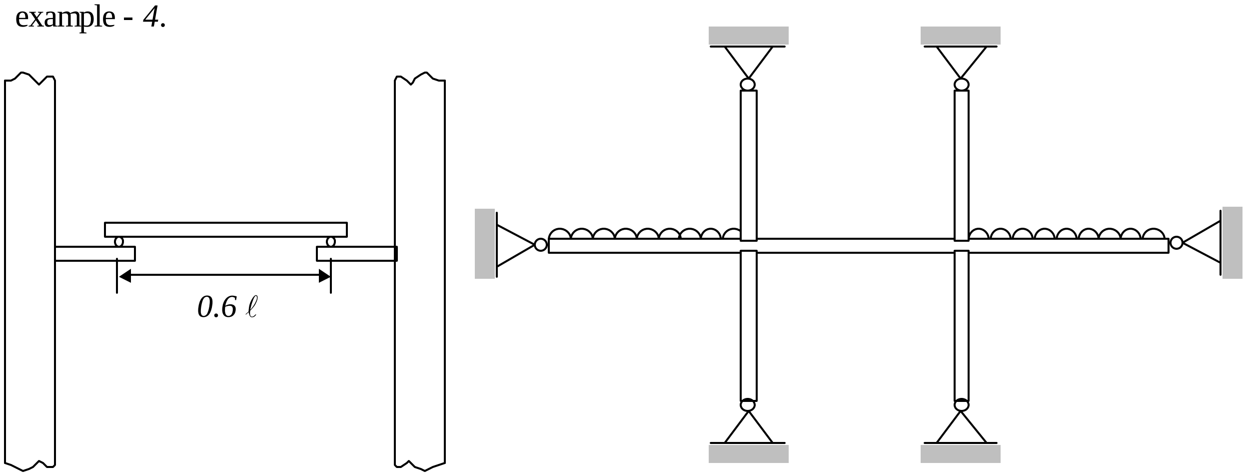
4.3.2 Substitute frame method

Rigid frame high-rise buildings are highly redundant structures. The analysis of such frames by conventional methods such as moment distribution method or Kane's method is very lengthy and time consuming. Thus, approximate methods (such as two cycled moment distribution method) are adopted for the analysis of rigid frames under gravity loading, one of such methods is Substitute Frame Method.

Substitute frame method is a short version of moment distribution method. Only two cycles are carried out in the analysis and also only a part of frame is considered for analysing the moments and shears in the beams and columns. The assumptions for this method are given below:

1) Moments transferred from one floor to another floor are small. Hence, the moments for each floor are separately calculated.
2) Each floor will be taken as connected to columns above and below with their far ends fixed.

If the columns are very stiff, no rotation will occur at both ends of a beam and the point of contraflexure will be at about $0.2\ \ell$. The actual beam can be replaced by a simply – supported beam of span $0.6\ \ell$ as shown in Fig. 7(*a*). If, the columns are flexible, then all the beams can be considered as simply supported of span ℓ as the beam – column joint will rotate like a hinge, an approximate model for middle floor beam is shown in Fig. 7(*b*). An approximate method of analysis for gravity loads is illustrated in worked example - *4*.



(a) Stiff column frame

(b) Flexible column frames

Fig. 7 Substitute approximate models for analysis of frames

4.4 Drift in Rigid Frames

The lateral displacement of rigid frames subjected to horizontal loads is due to the following three modes:

- Girder Flexure
- Column Flexure
- Axial deformation of columns

The sum of the storey drifts from the base upward gives the drift at any level and the storey drifts can be calculated from summing up the contributions of all the three modes discussed earlier in that particular storey. If the total drift or storey drift exceeds the limiting value then member sizes should be increased to avoid excessive drift.

5.0 COMPUTER ANALYSIS OF RIGID FRAMES

Although the approximate methods described earlier have served structural engineers well for decades, they have now been superseded by computer analysis packages. Computer analysis is more accurate, and better able to analyse complex structures. A typical model of the rigid frame consists of an assembly of beam-type elements to represent both the beams and columns of the frame. The columns are assigned their principal inertia and sectional areas. The beams are assigned with their horizontal axis inertia and their sectional areas are also assigned to make them effectively rigid. Torsional stiffnesses and shear deformations of the columns and beams are neglected.

Some analysis programs include the option of considering the slab to be rigid in plane, and some have the option of including P-Delta effects. If a rigid slab option is not available, the effect can be simulated by interconnecting all vertical elements by a horizontal frame at each floor, adding fictitious beams where necessary, assuming the beams to be effectively rigid axially and in flexure in the horizontal plane.

6.0 SUMMARY

In this chapter short cut methods for approximate analyses of simple braced frame as well as for frames with moment resisting joints are described and illustrative worked examples appended. Simplified analyses of building frames with gravity loads as well as frames with lateral loads have been discussed. More accurate methods making use of flexibility or stiffness matrices are generally incorporated in sophisticated software in many design offices.

7.0 REFERENCES

1. McCormac. C. J., (1994): "Structural Steel Design", Harper Collins College Publishers.
2. Owens. G. W. and Knowles. P., (1994) "Steel Designers Manual", The Steel Construction Institute, ELBS Blackwell Scientific Publishers, London.
3. Taranath. S. B., (1984): "Structural analysis and design of tall buildings", McGraw-Hill Book Company
4. Schuller. W., (1976): "High-rise building structures", John Wiley & Sons
5. Smith. B. S., and Coull. A., (1991): "Tall building structures: Analysis and Design", John Wiley & Sons.